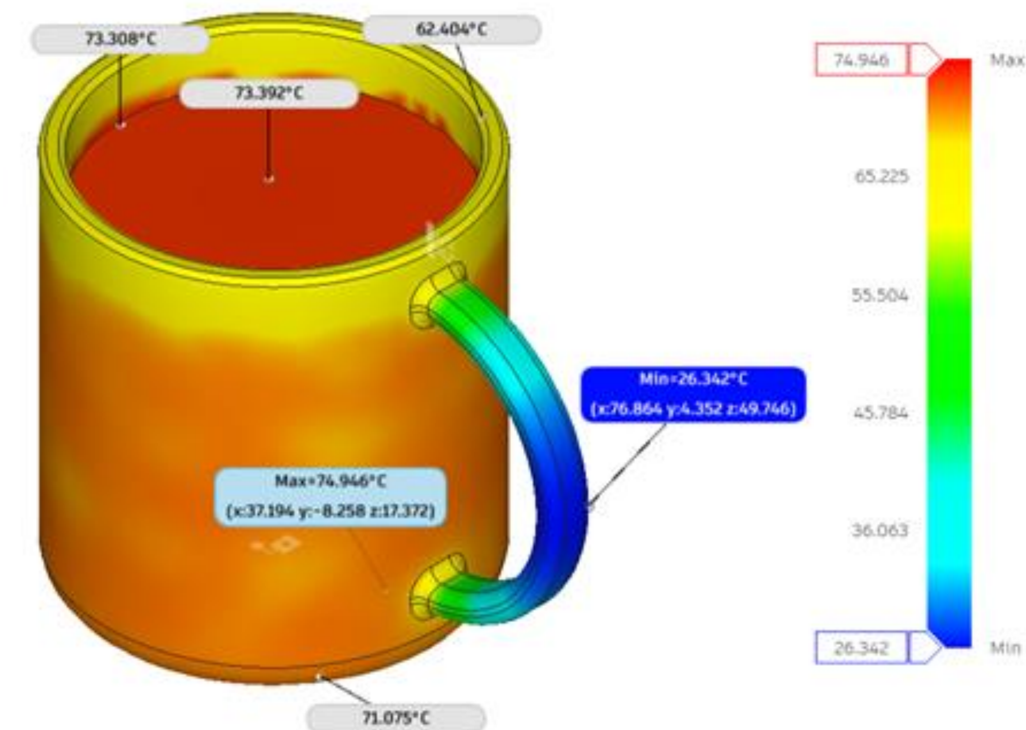
FEA Thermal Simulation using Fusion

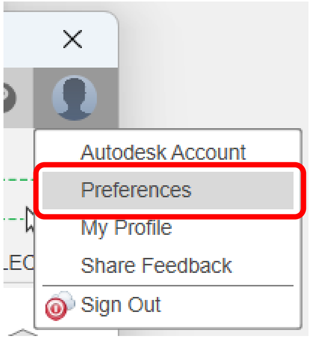
My pretty FEA results should be worth an art credit.



Today’s lesson is sponsored by Diggerland in West Berlin, NJ

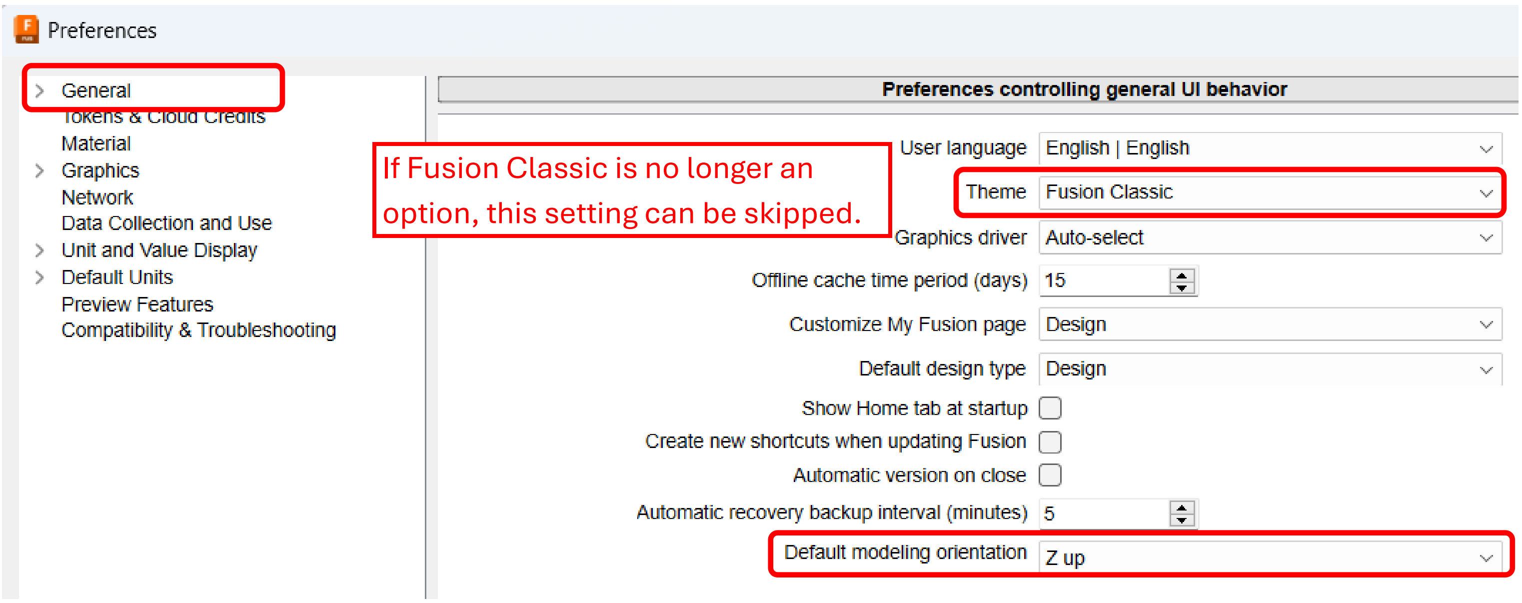
Contents

Setting Fusion Preferences

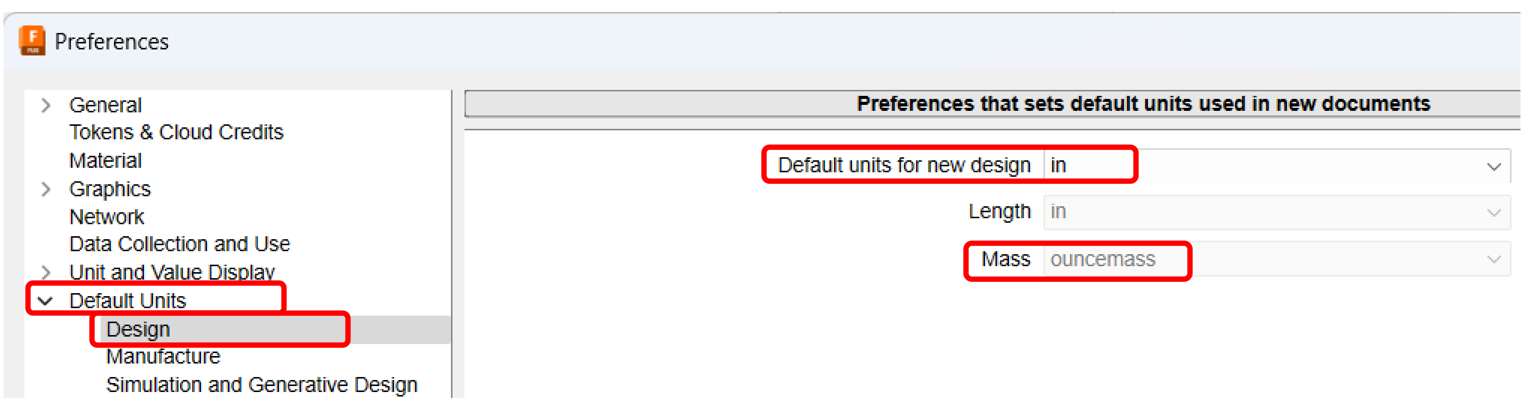


- if this is your first time using Fusion open Preferences by clicking on the top right icon and selecting Perferences

- in the **General** section select **Fusion Classic** for the **Theme** and **Z up** for **Default modelling orientation**



- in the **Default Units** section select **Design** and change the **Default units** to **in** and **Mass** to **ouncemass**
- click the bottom right **Apply** button and then the **OK** button

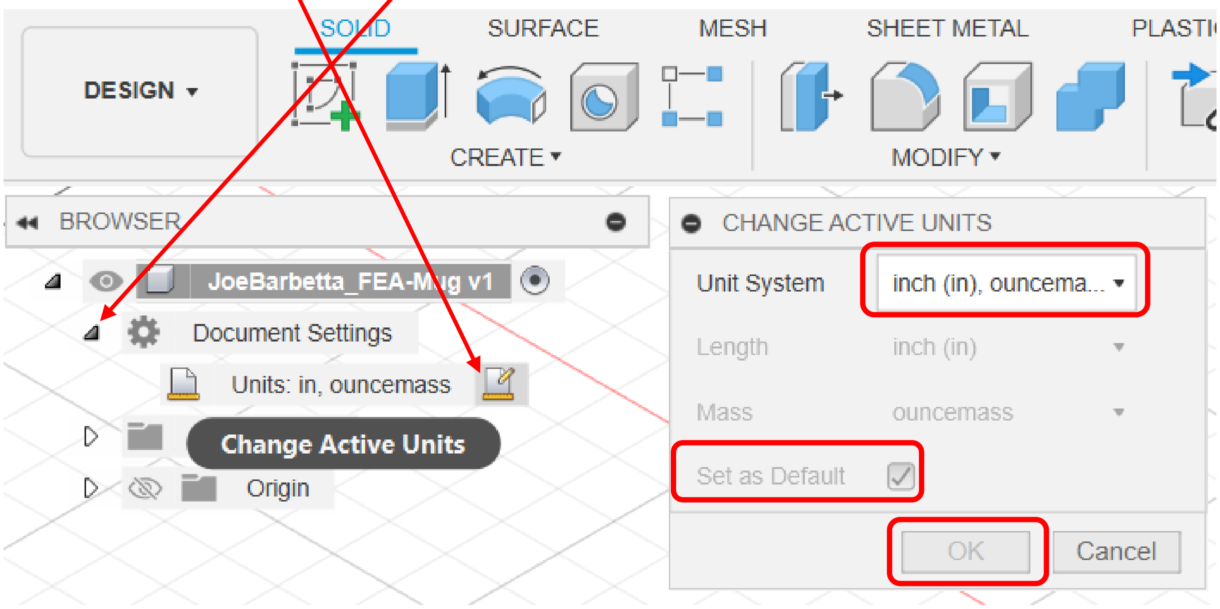


- click on the **x** to close the project, which will start a new project. This is done to allow new Preferences to be applied to the new project.

Starting a Design in Fusion

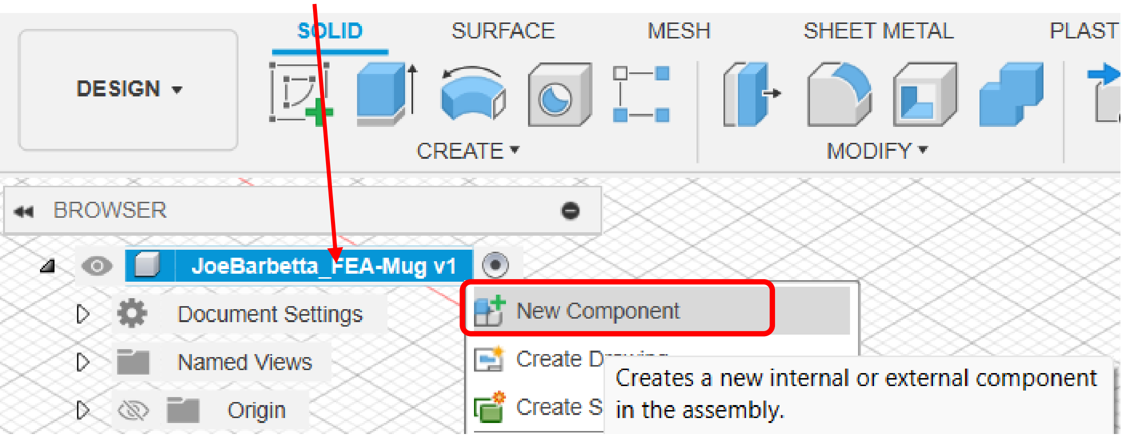
- open **Fusion**. If there is no icon on the Desktop, use the Windows search (magnifying glass icon) and type fusion
- from top **File** icon select **Save** and name the file.
  Use your name followed by _ **FEA-Mug** e.g. **JoeBarbetta_FEA-Mug** (note the use of the underscore)

- in the left **"BROWSER"** click the **arrow next to Document Settings**
- click on the **edit icon** that appears to the left when you hover over **Units**
- ensure **Active Units** are set to **Units: in, ouncemass** and click **OK**. You can also enable **Set as Default** if it is not grayed out.

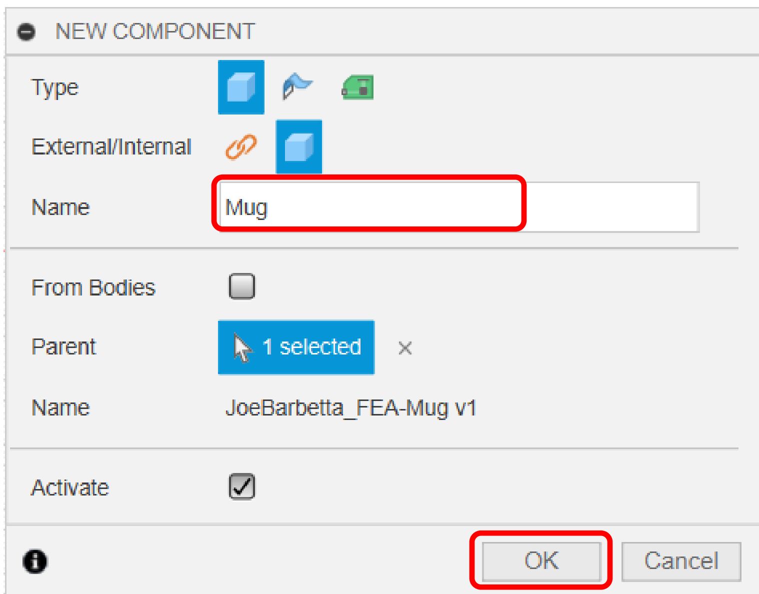


Creating the Profile Sketch for the Mug

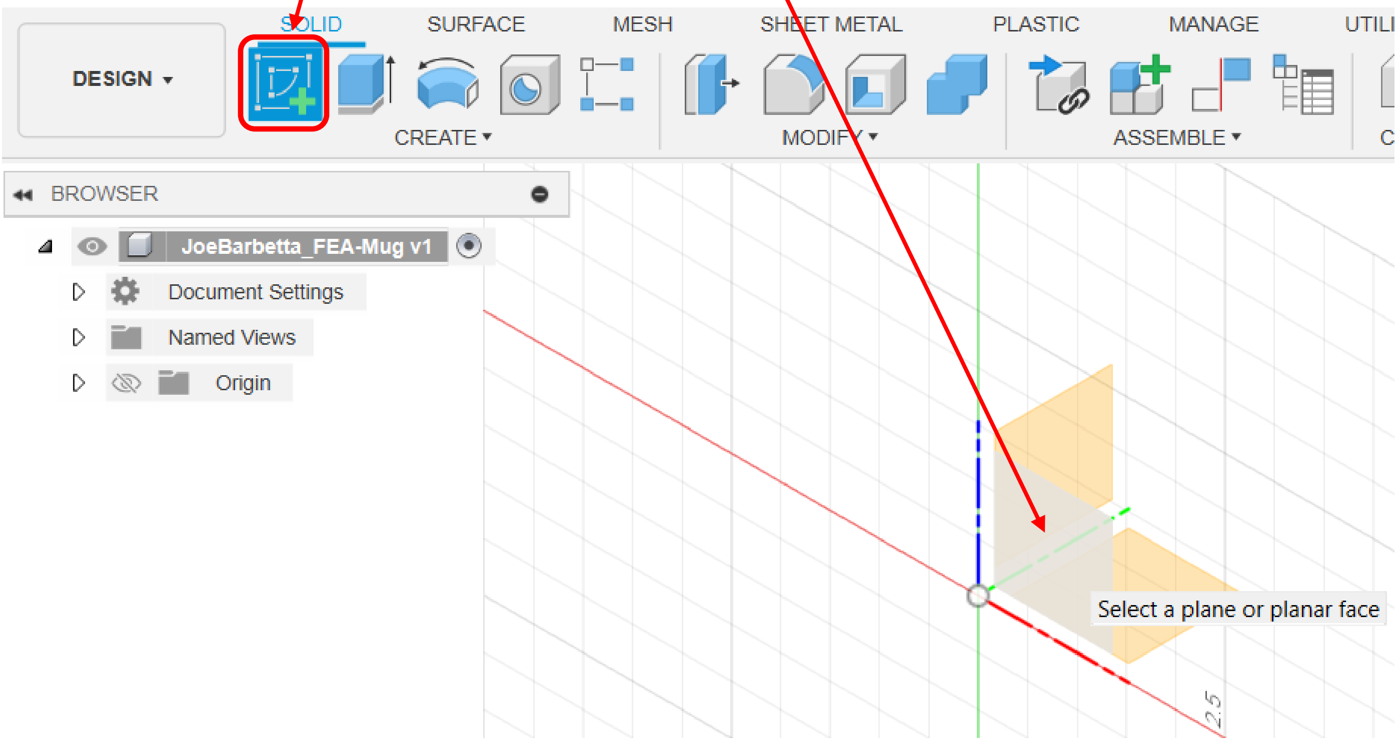
- right-click on the **project name** and select **New Component**



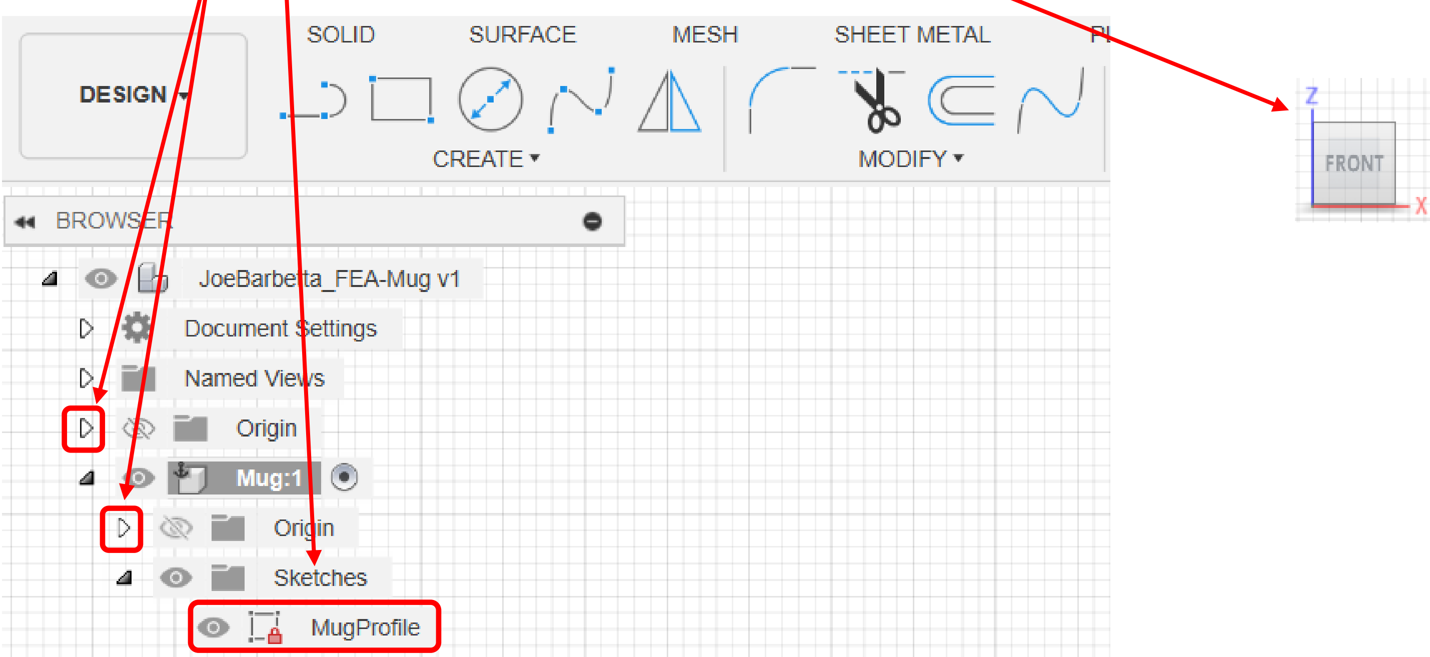
- name the component **Mug** and click **OK**



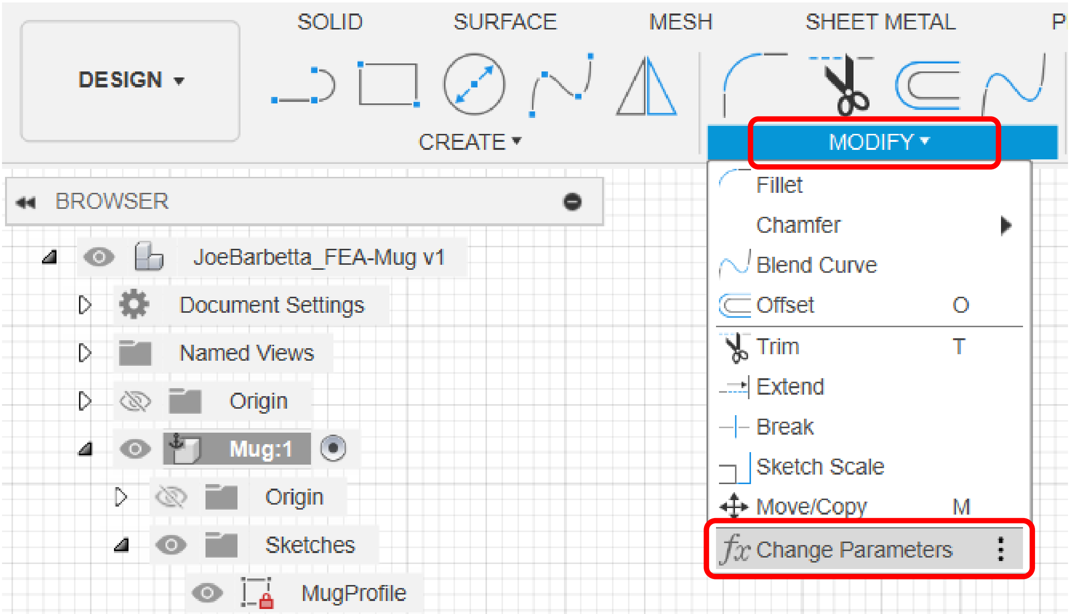
- select the top **Create Sketch** tool and click on the **front rhombus** to select the X-Z Plane.
If the Create Sketch tool can't be found, find it within the **CREATE** menu.



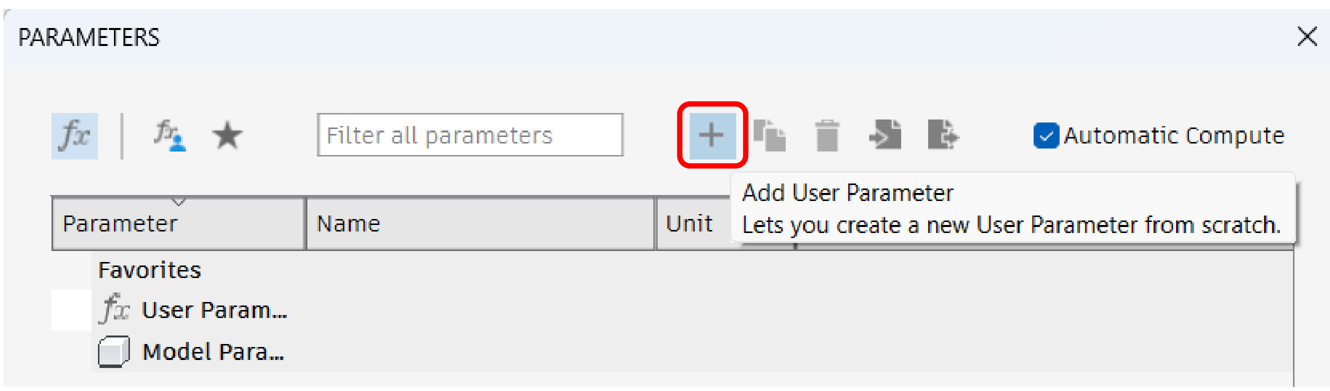
The **View Cube** at the top right text should show "**FRONT**". If it does not, then redo the Create Sketch and ensure the correct rhombus is selected.
- click on the **arrow** next to the **Component Name** and then next to **Sketches** folder to view any sketches
- right-click on the **Sketch** and select **Rename** and rename it **MugProfile**.



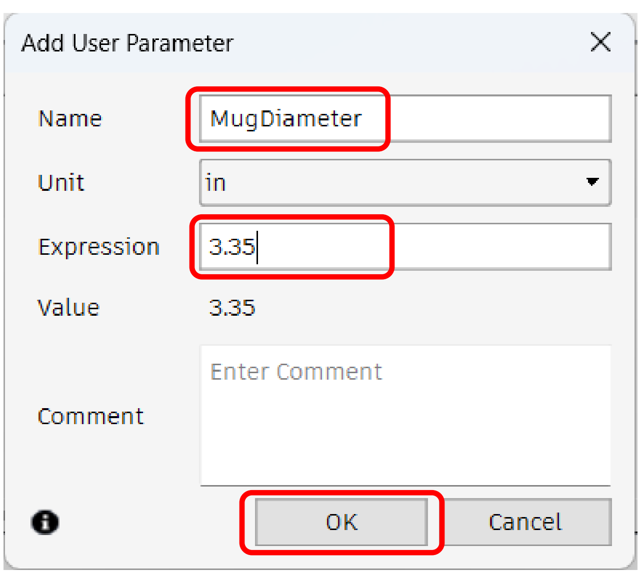
- from the **MODIFY** menu select **Change Parameters**



- click the **+** icon



- for the **Name** enter **MugDiameter** and for the **Expression** enter **3.35** and click **OK**

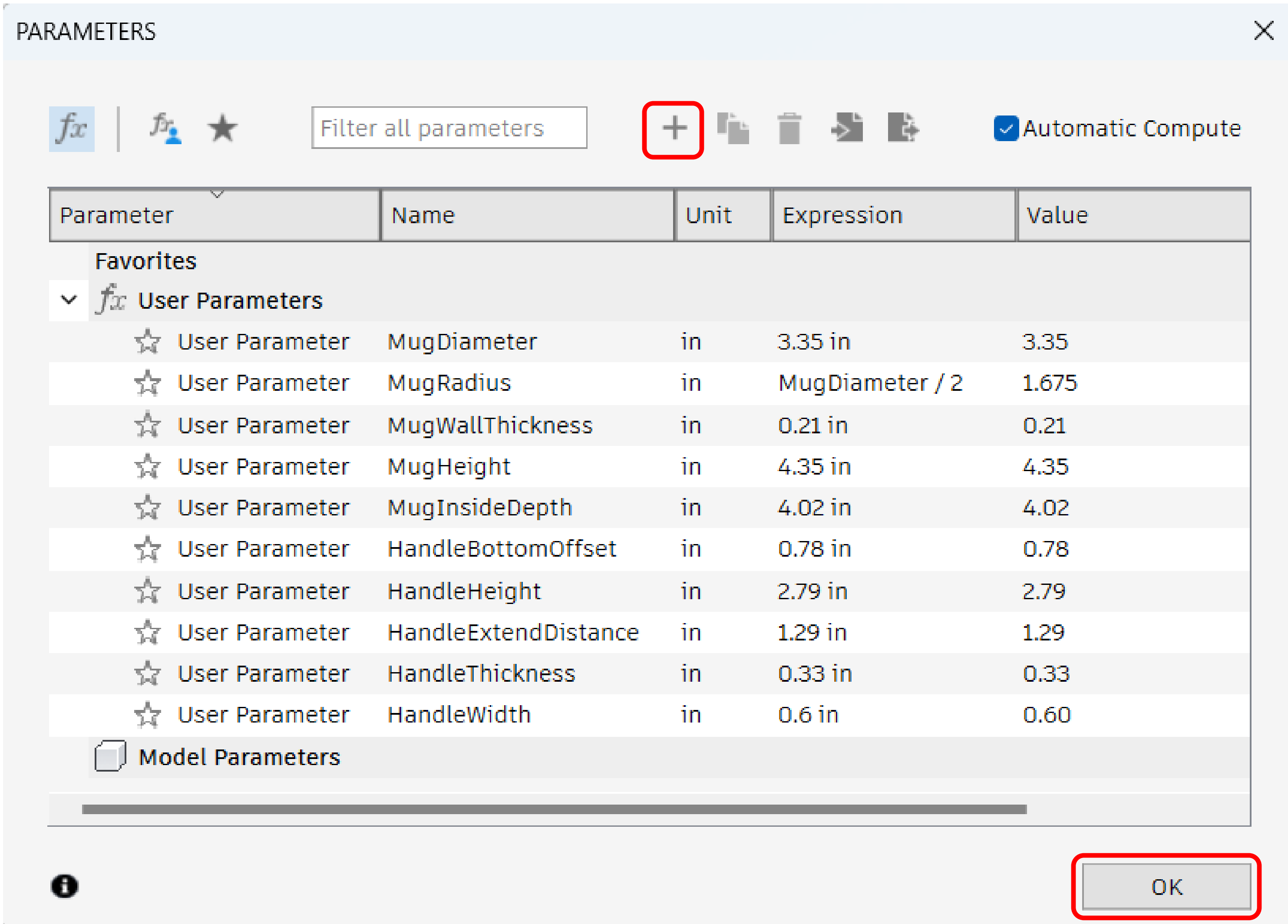


Note that Fusion calls the entry box **Expression** and below it shows a **Value**.

One can enter an equation based on other Parameters and the resultant Value is calculated.

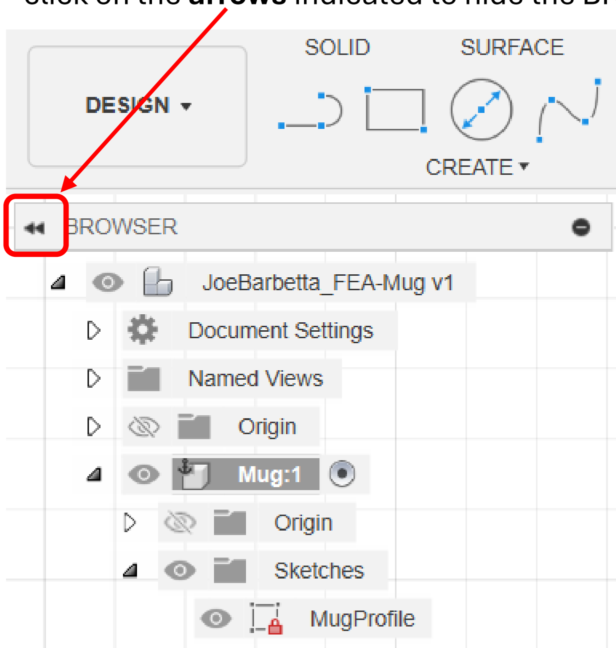
- continue using the **+** icon to enter **MugRadius and the other values** as shown. Note that MugRadius has an equation based on the first parameter divided by 2.

- click **OK** after all have been entered

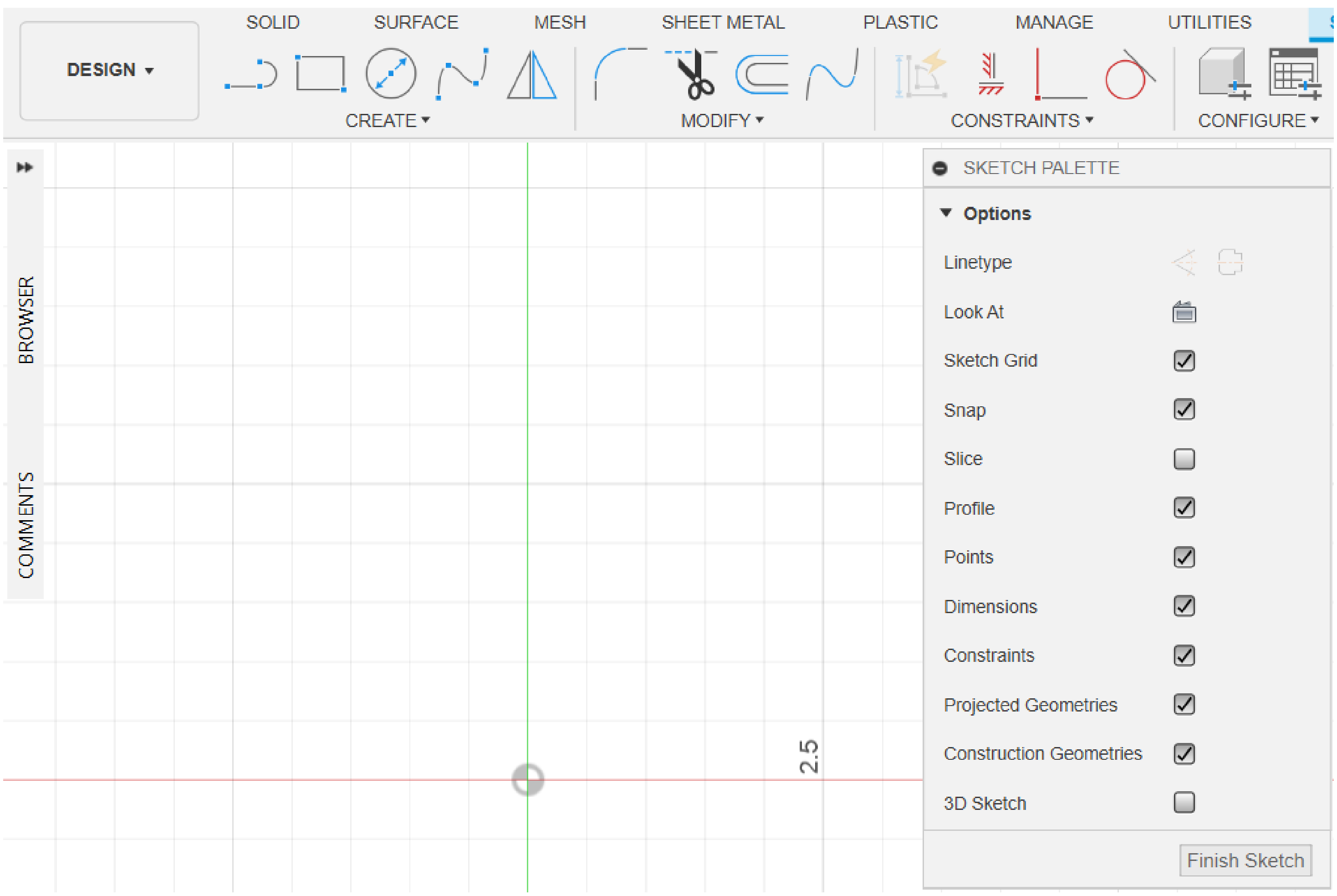


- if you enjoyed entering all these parameters, close your project without saving and enter them again

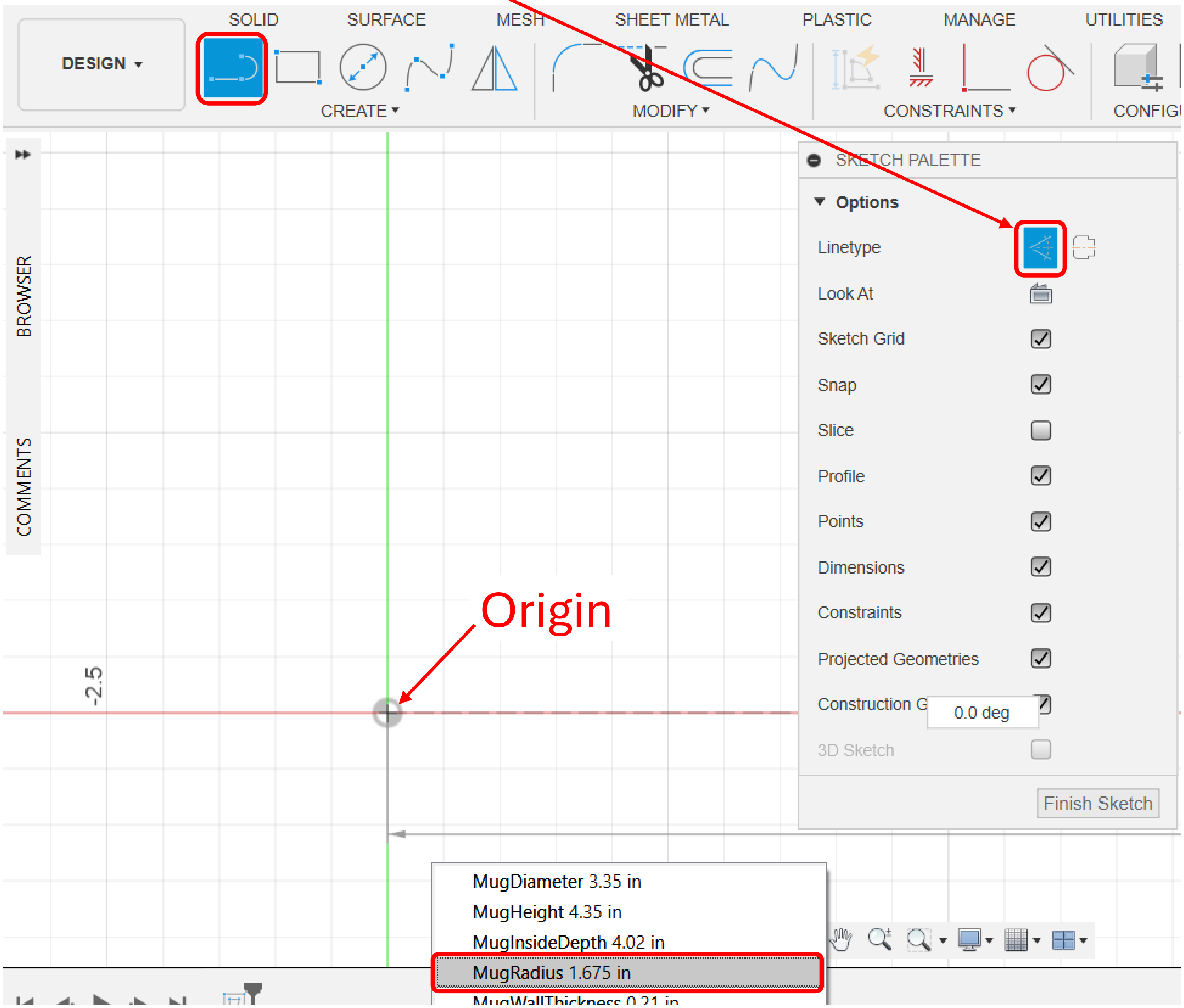
- click on the **arrows** indicated to hide the BROWSER. It can be reopened at any time using the same arrows.



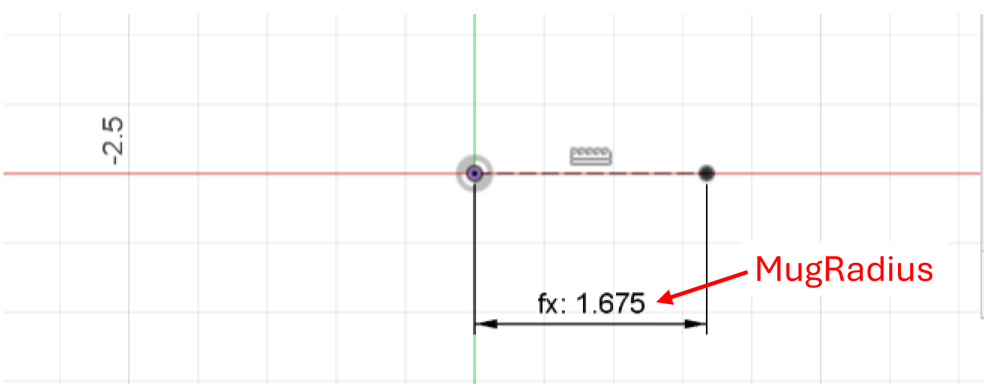
- turn the **Mouse wheel** to zoom and hold the **mouse wheel down** to pan to achieve a view as below



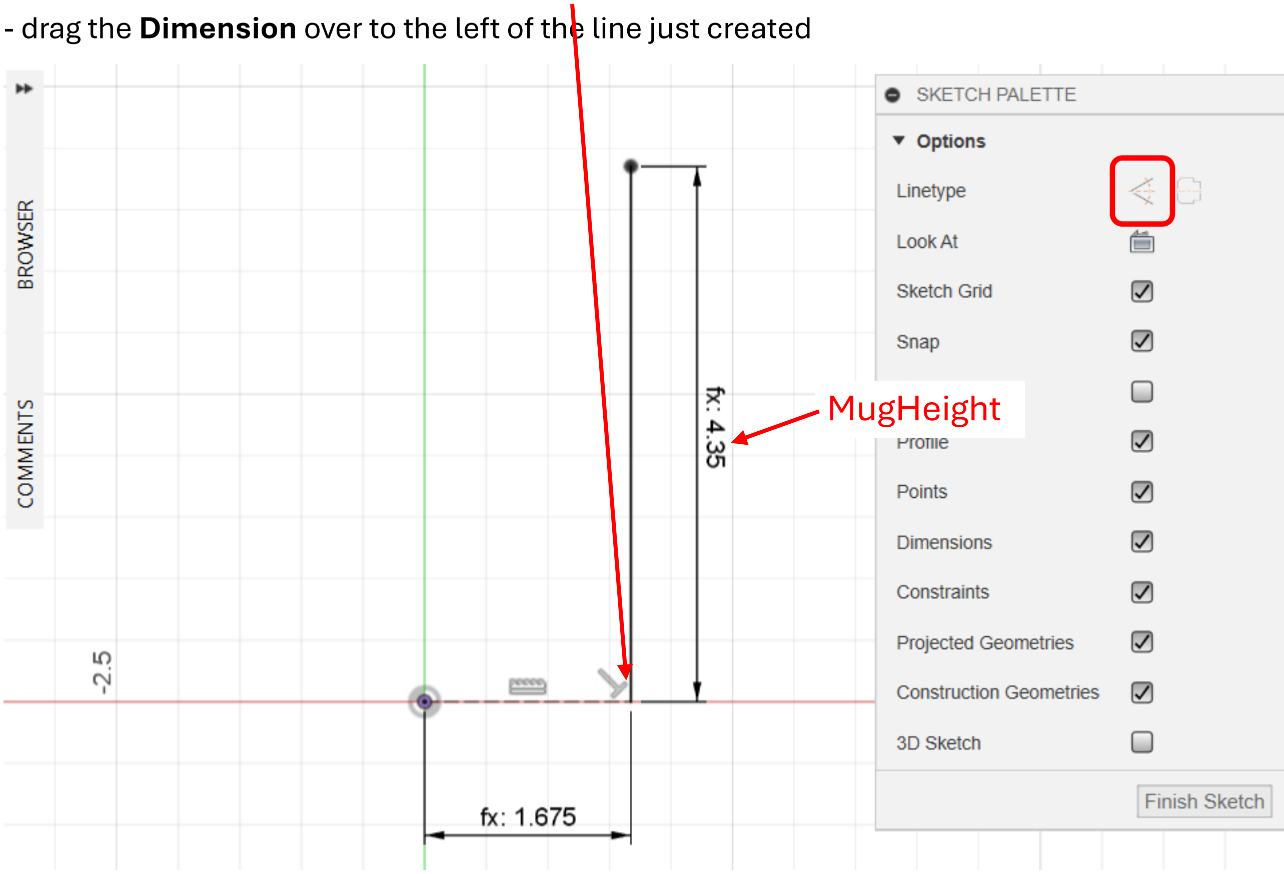
- select the **Line** tool (if it is not visible find it in the CREATE menu) and click on the **Origin** to start the line
- click on the **Construction** line icon to highlight it blue
- click on the **Origin** to start the line, extend the line to the right, type **m**, which should show a parameter list, use an **arrow key to select MugRadius**, and press the **Enter key**. Do not click Finish Sketch.



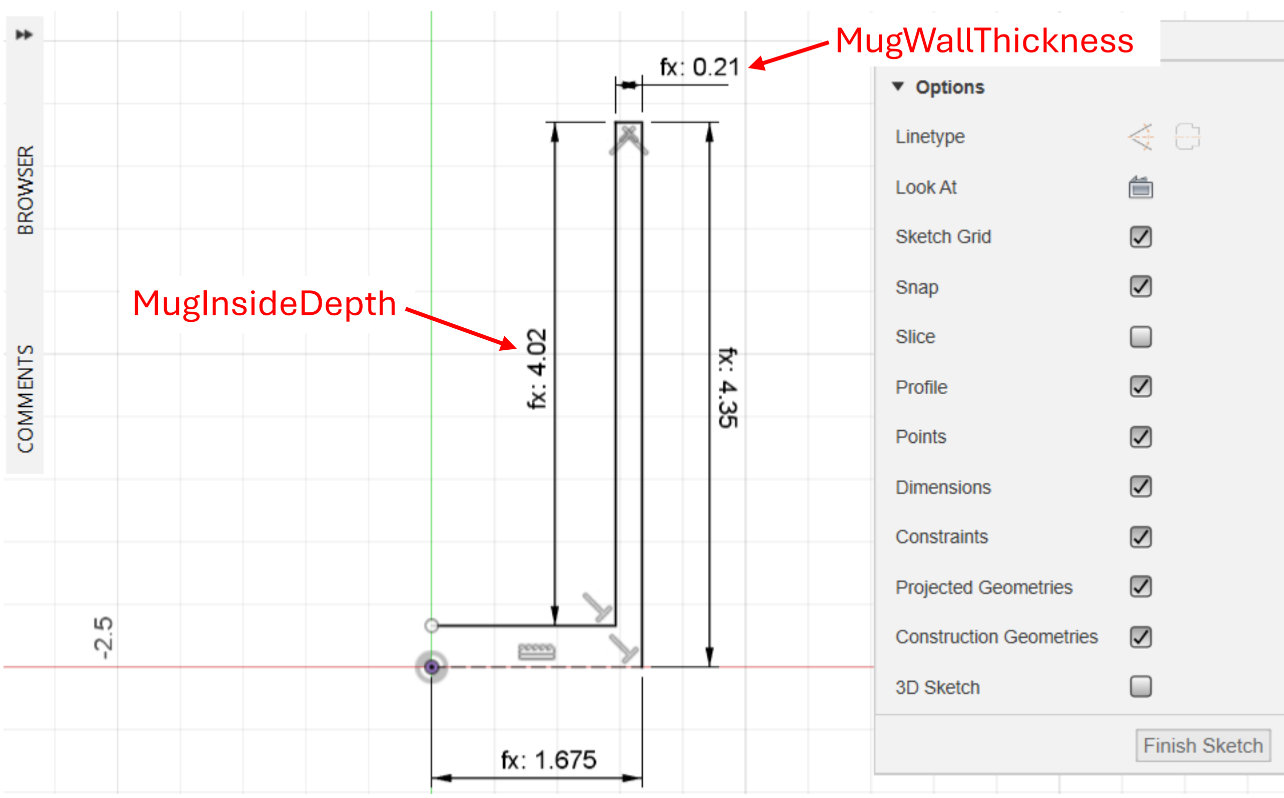
The resulting line should look like that below.



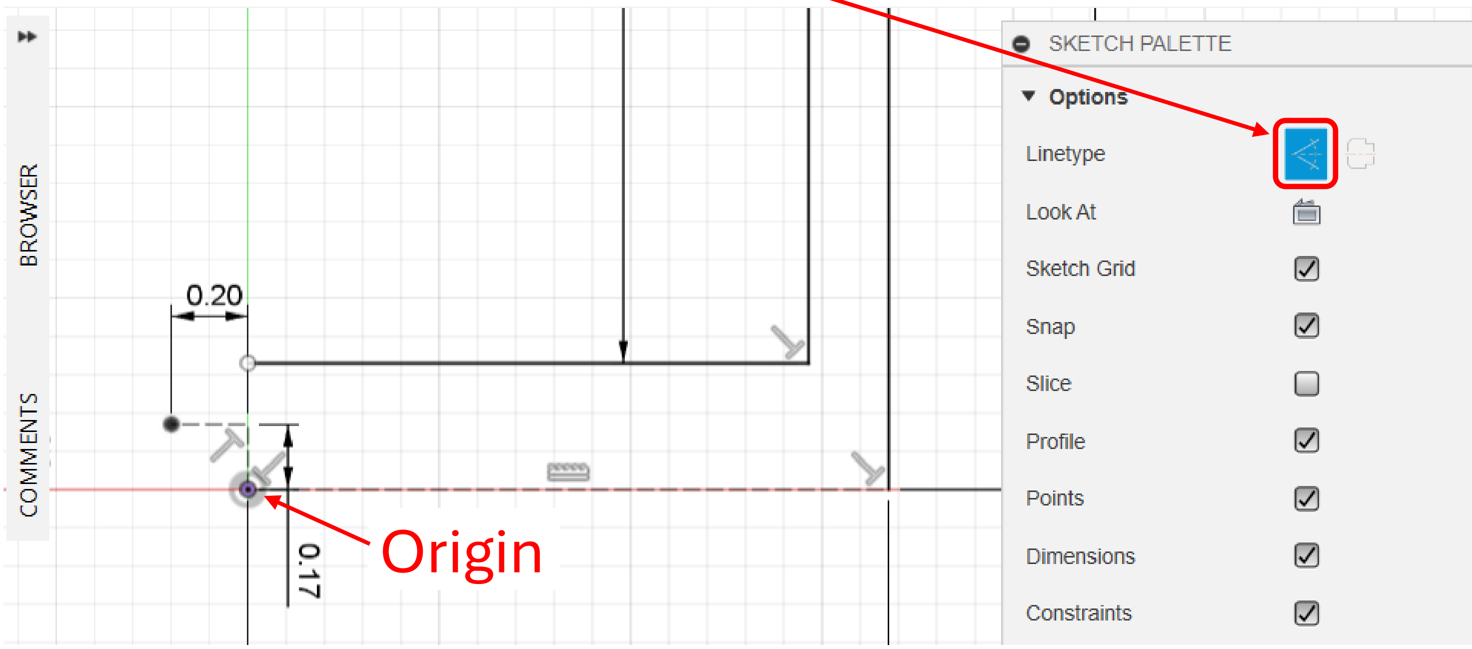
- click on the **Construction** line icon to remove the blue highlighting
- use the **Line** tool and start a line at the **end of the bottom line** and upward using the **MugHeight** parameter
- drag the **Dimension** over to the left of the line just created



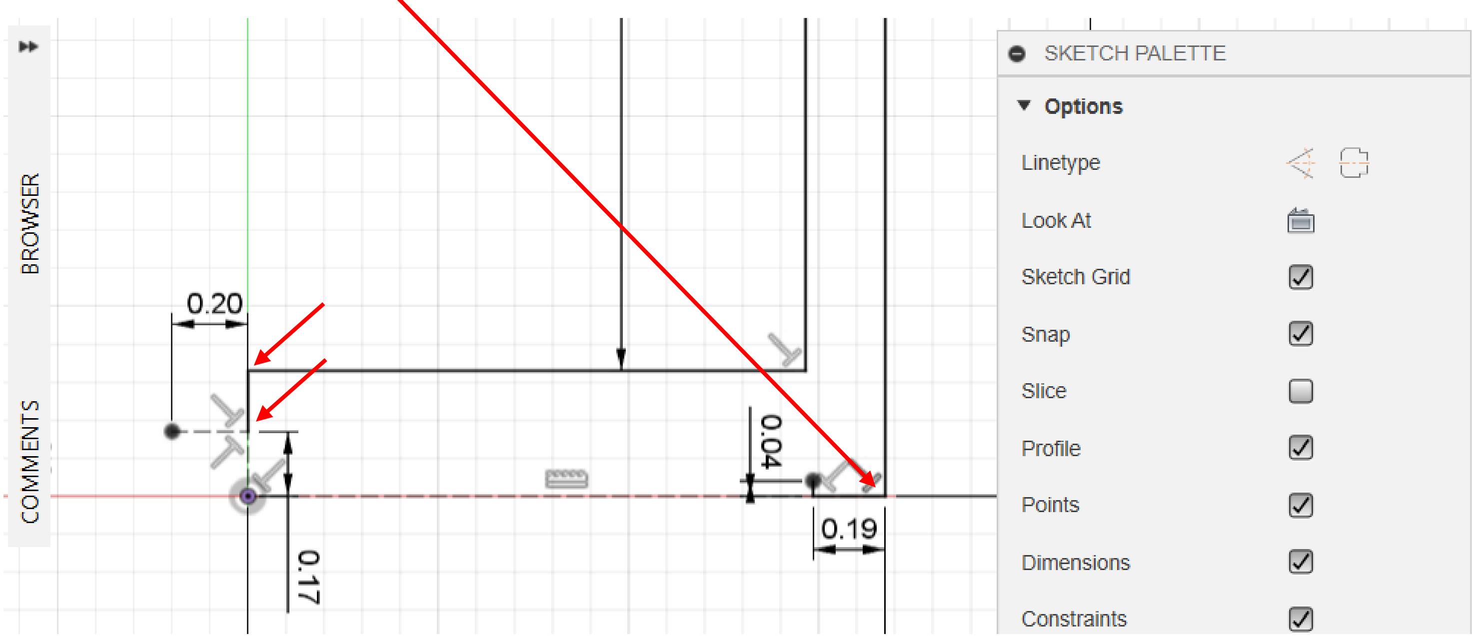
- from the top of the vertical line, create a line **extended to the left using MugWallThickness**, and then another line **extended downward using MugInsideDepth**
- create a line from that **last point and over to the left to the axis**



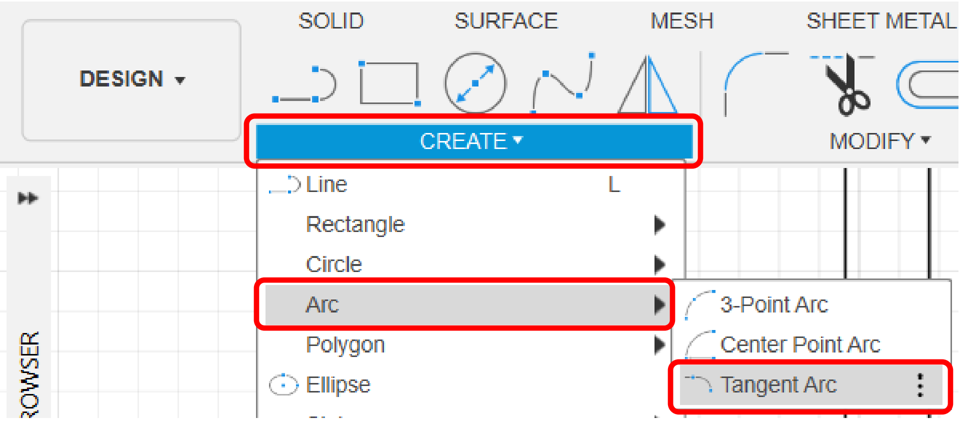
- zoom in to the bottom of the sketch
- click on the **Construction** line icon to highlight it blue
- start a line from the **origin** and up by **0.17**. Here this value is being typed into the dimension box instead of selecting a parameter.
- from that point create a line to the left using **0.20**



- click on the **Construction** line icon to remove the blue highlighting
- yell **"I am tired of clicking that on and off!"**
- from the **bottom right corner** create a **line extended to the left using 0.19** and then **another line upward using 0.04**
- create a line **connecting the two points** indicated by the arrows

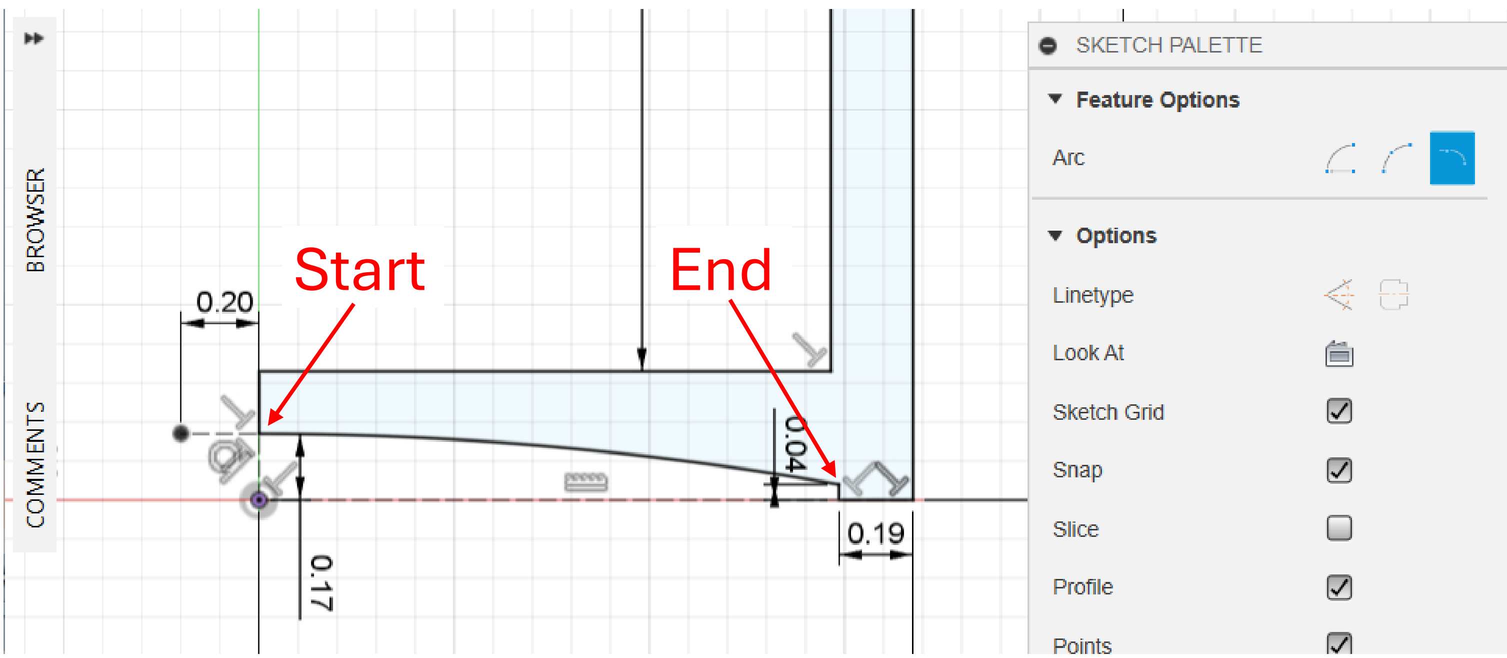


- from the **CREATE** menu select **Arc** and **Tangent Arc**

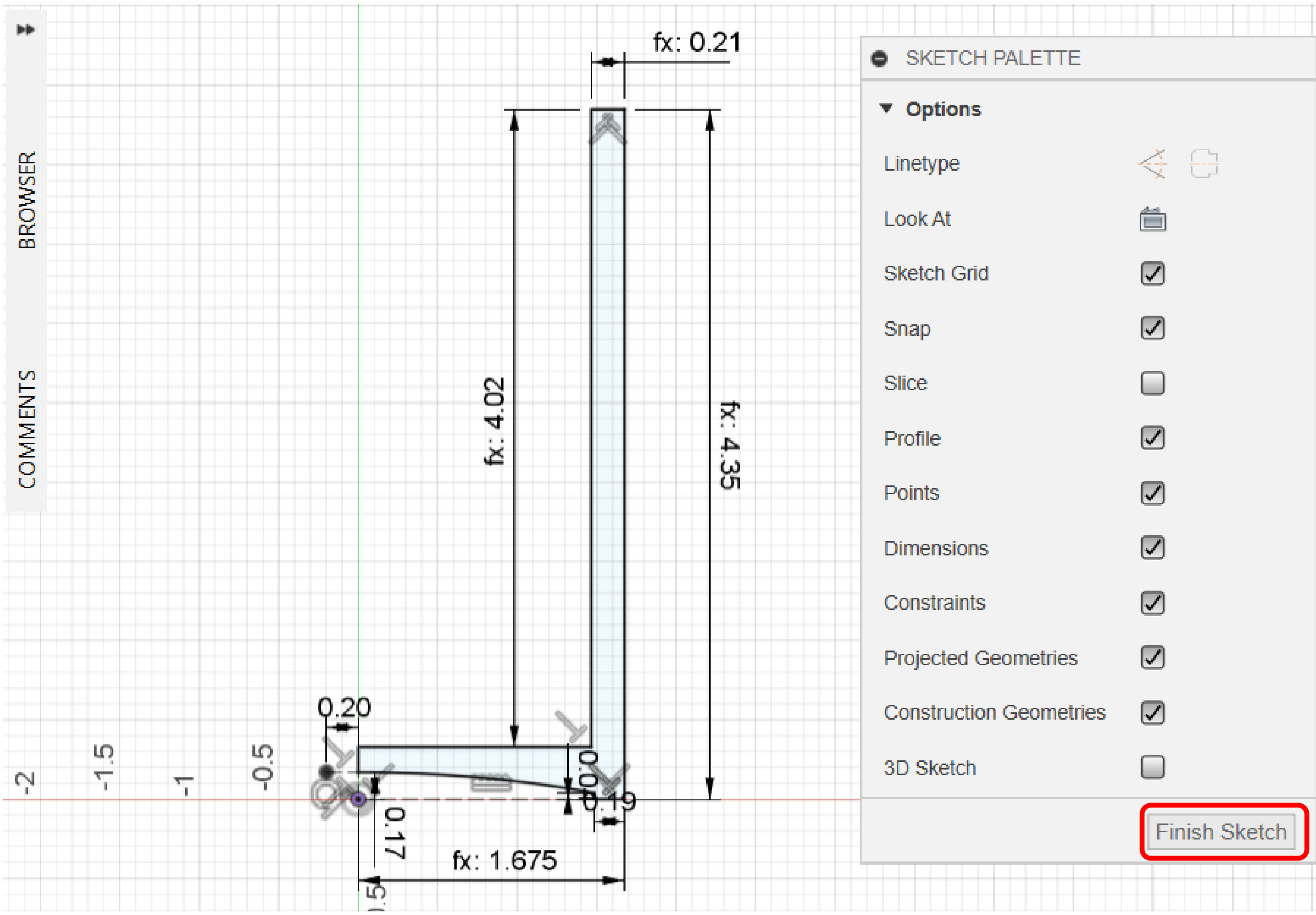


- start the **arc from the point indicated on the left to that indicated on the right**, which should result in the closed section turning light blue

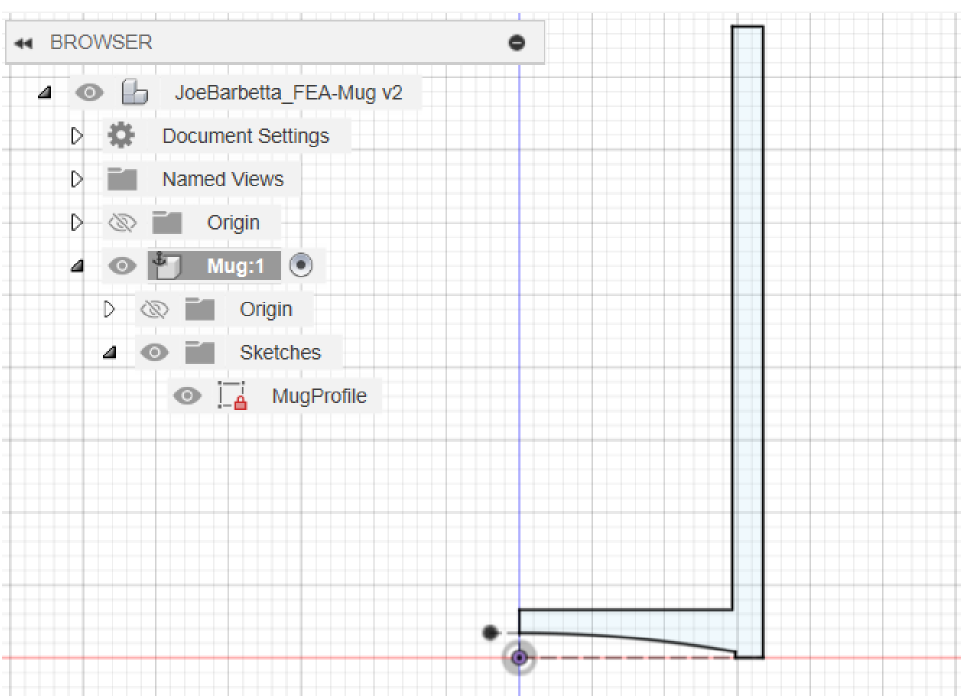
The purpose of the 0.20 length line on the left was just to provide a line for the Tangent Arc to be tangent from. A length of 0.20 was arbitrary for this line.



- zoom out to see the entire sketch. The positions of the Dimension lines are not important, but they can be dragged to make them easier to see.
- yell **"That's a beautiful sketch!"** and click the **Finish Sketch** button



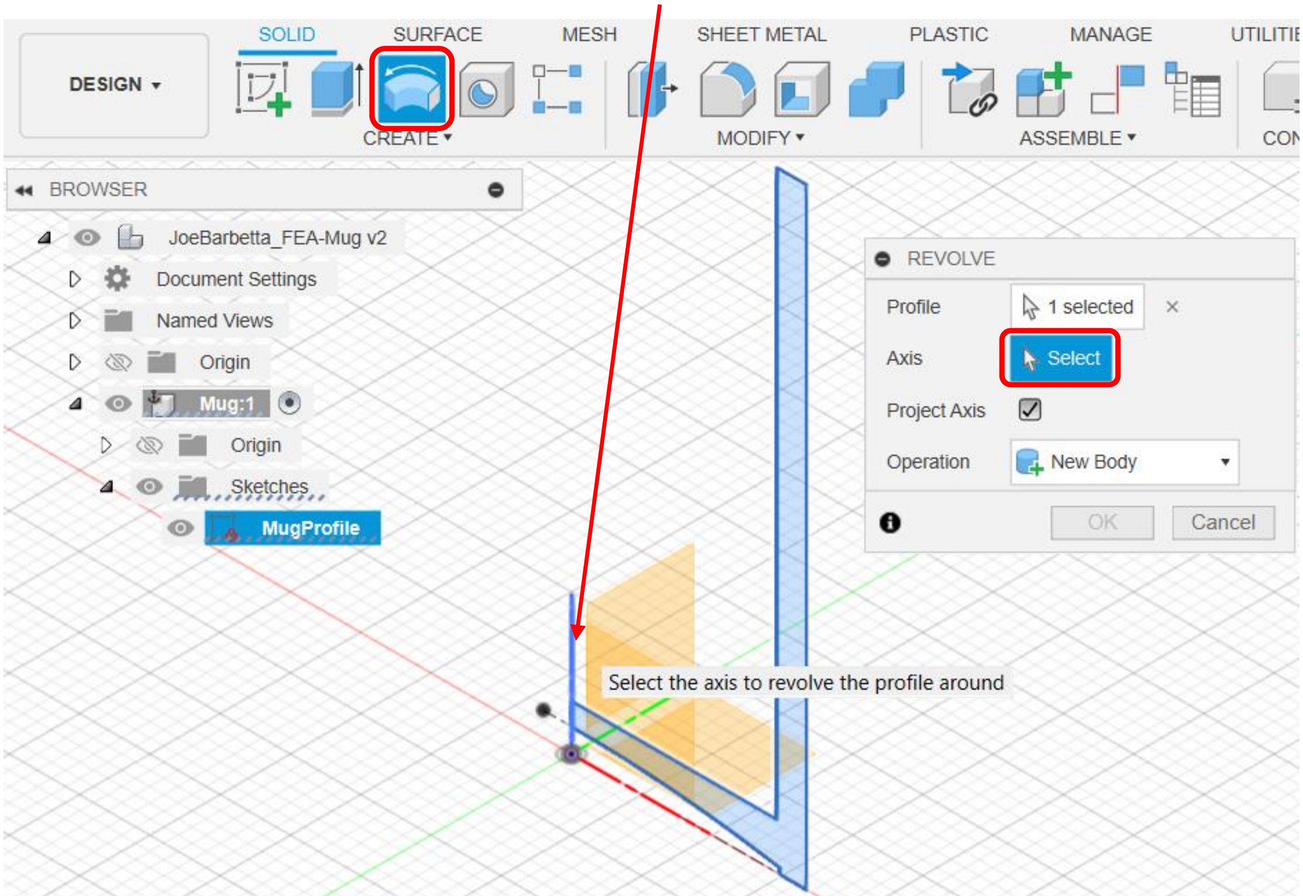
This is what it should look like with the BROSWER open.



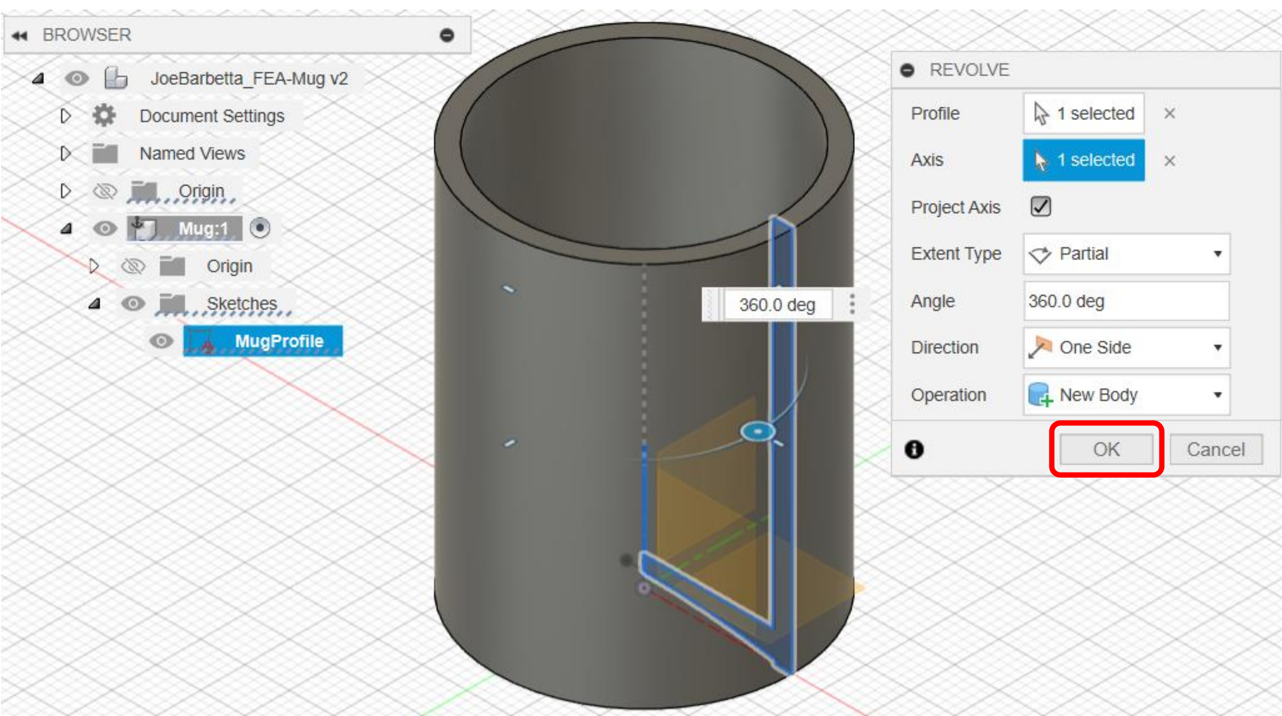
- click on the **Home** icon at the **View Cube**
- select the **Revolve** tool. If it is not visible find it in the CREATE menu.

Normally one would click on the blue profile region, but since this is the only profile now, it does so automatically.

- click on **Select** for **Axis** and click on the **vertical axis line**

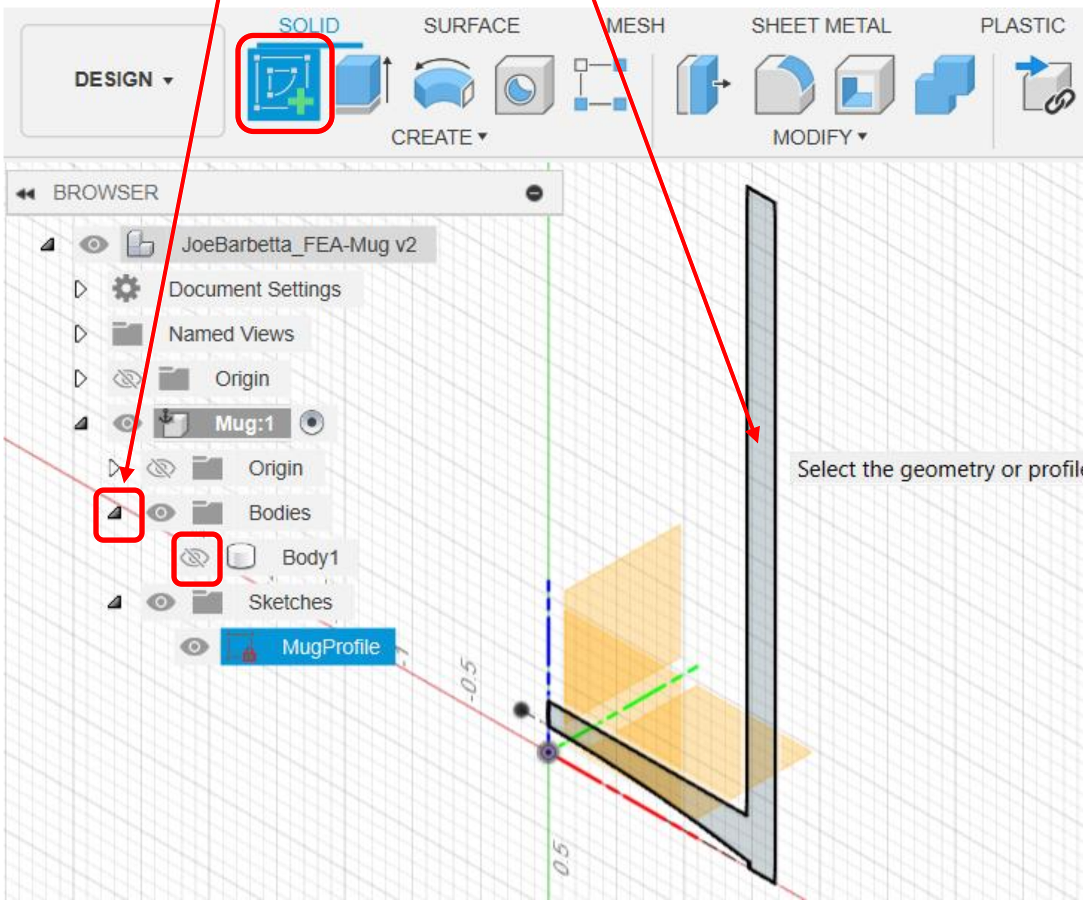


- zoom out to see the full mug body and yell "That was neat!" and click **OK**

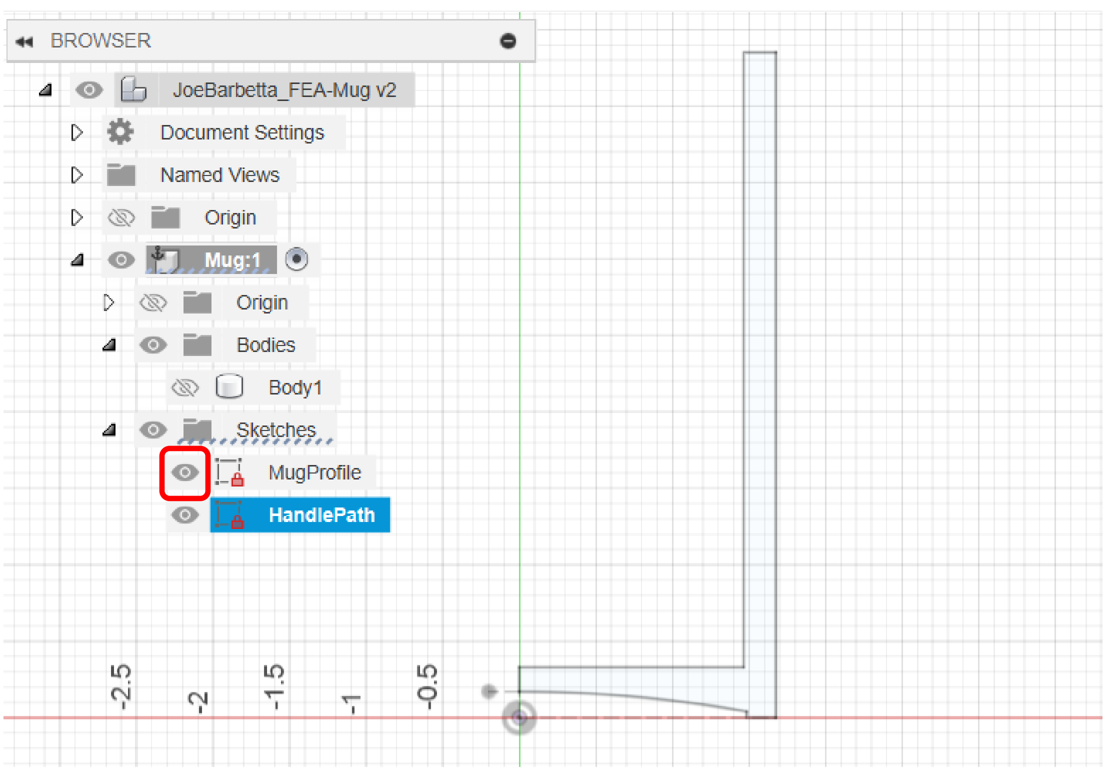


Creating the Handle Sketch

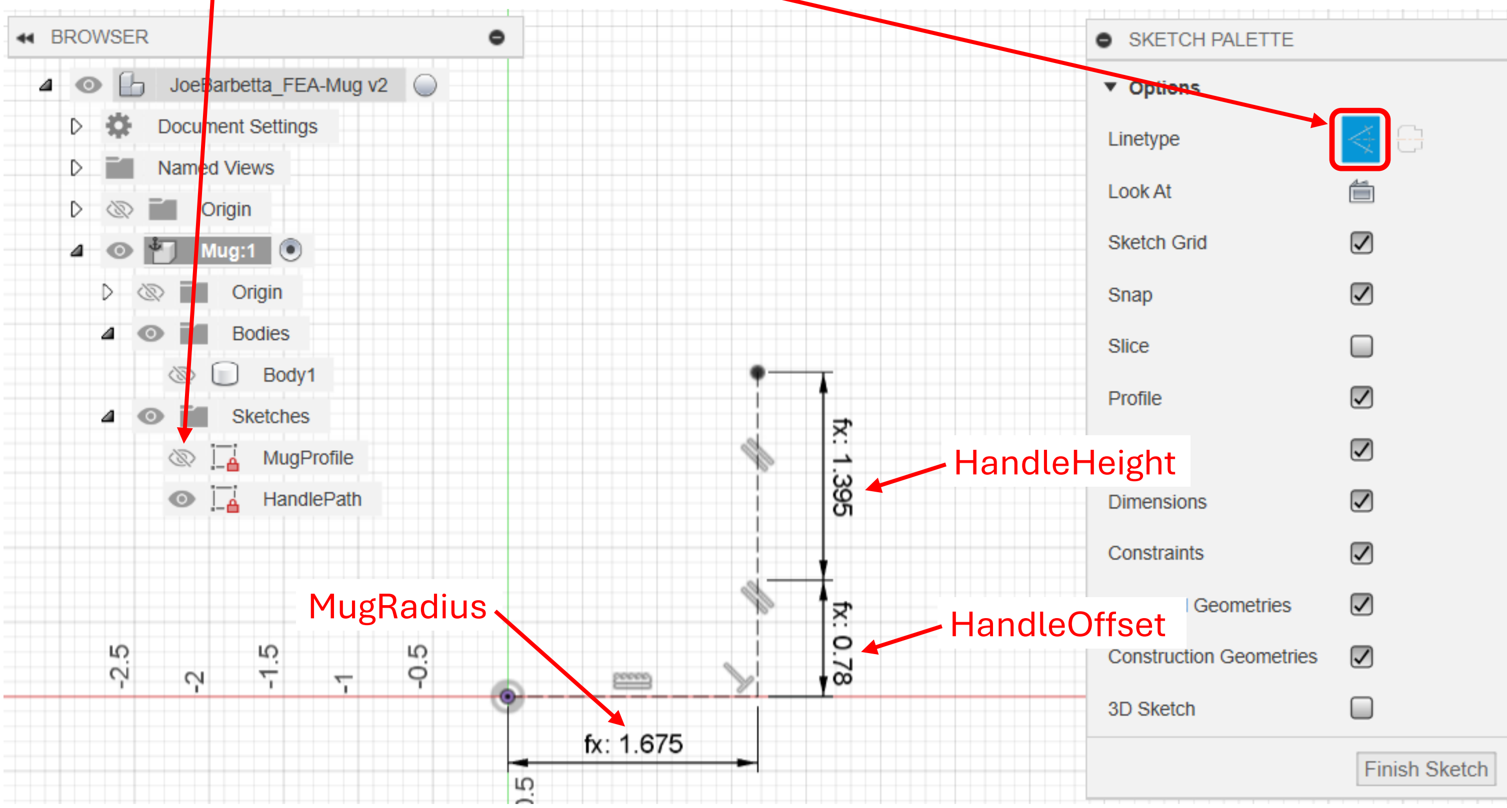
- click on the **arrow** to open the **Bodies** folder and click on the **eye** icon to hide the mug body
- select **Create Sketch** and click on the **profile**



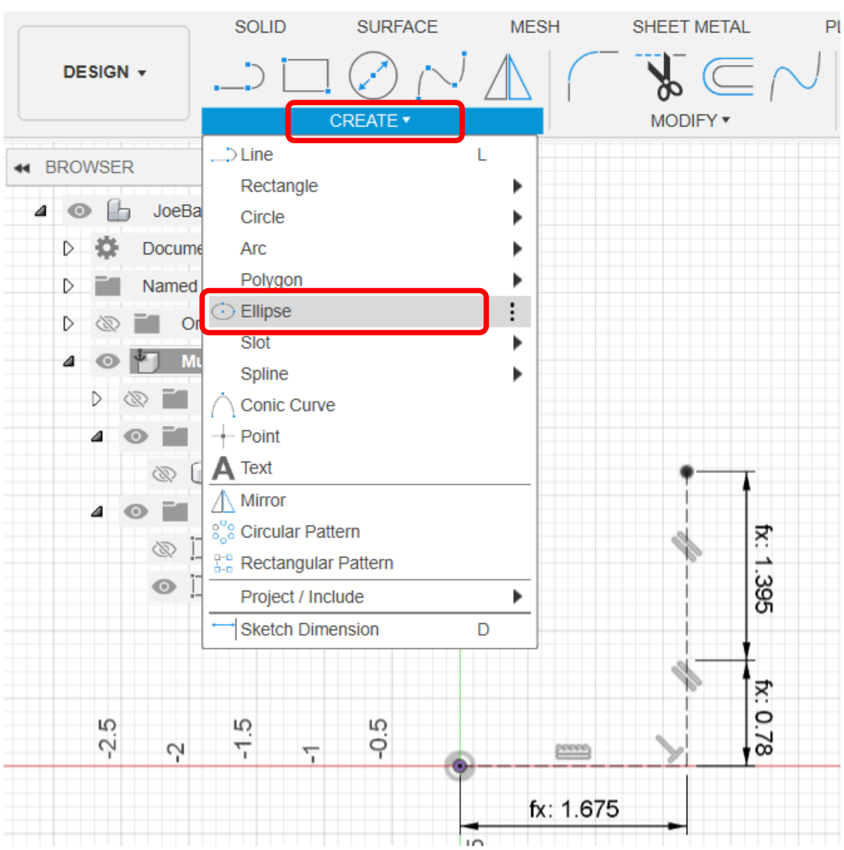
- rename this Sketch to **HandlePath** and click on the **eye** icon for **MugProfile** to make it visible



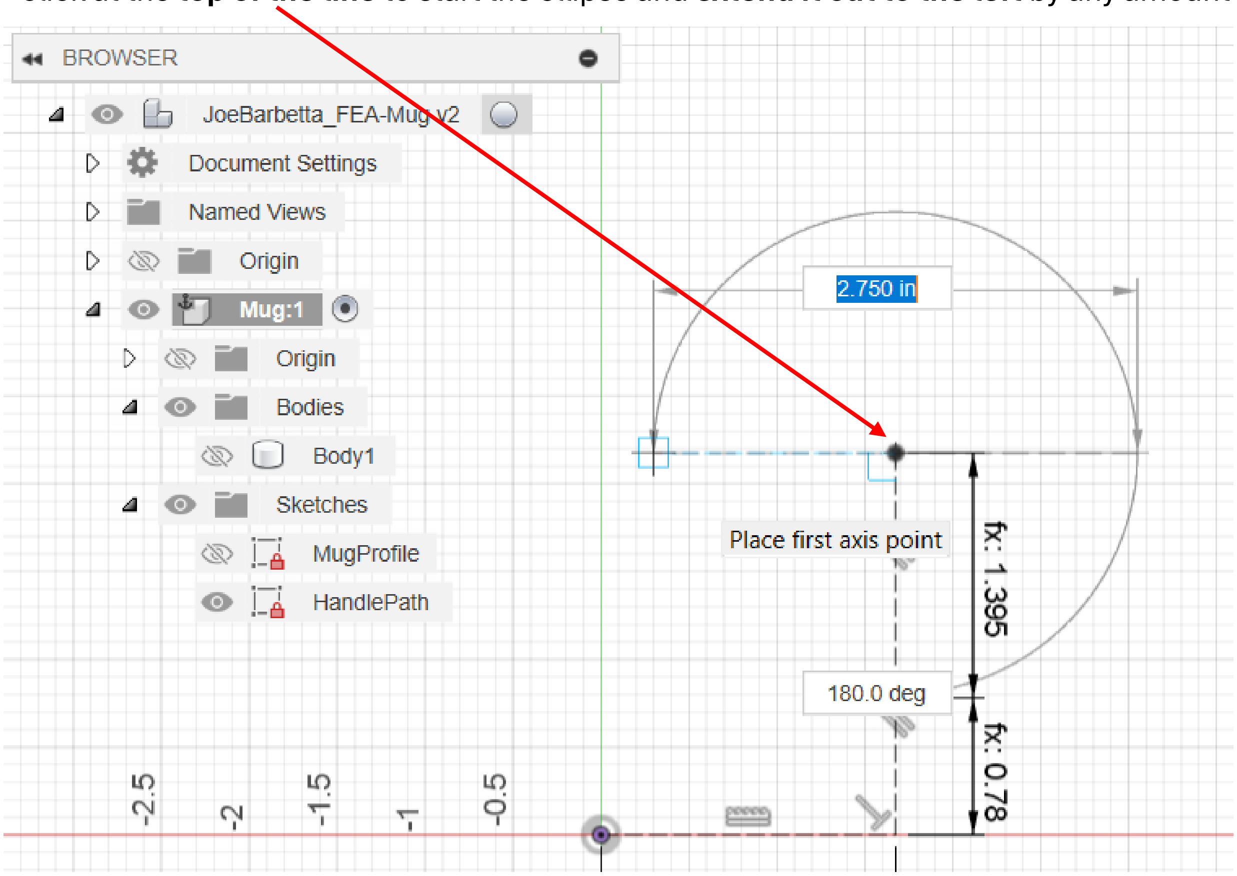
- click on the **eye** icon for **MugProfile** to hide it
- click on the **Construction** line icon to highlight it blue. Yell **"Here we go again!"**
- from the **Origin** create a line extended to the right using **MugRadius**, then a line upward using **HandleBottomOffset**, and then another line upward using **HandleHeight / 2**



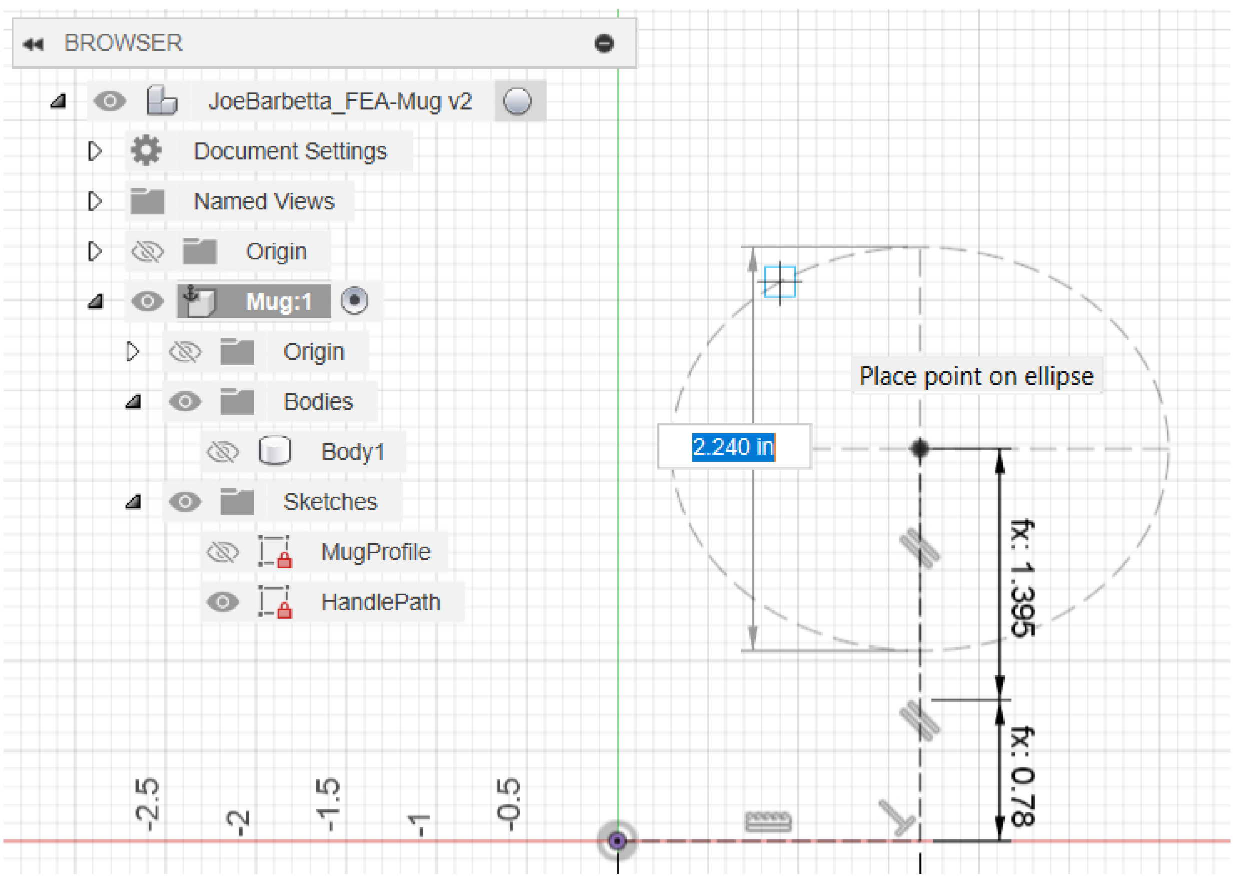
- from the **CREATE** menu select **Ellipse**



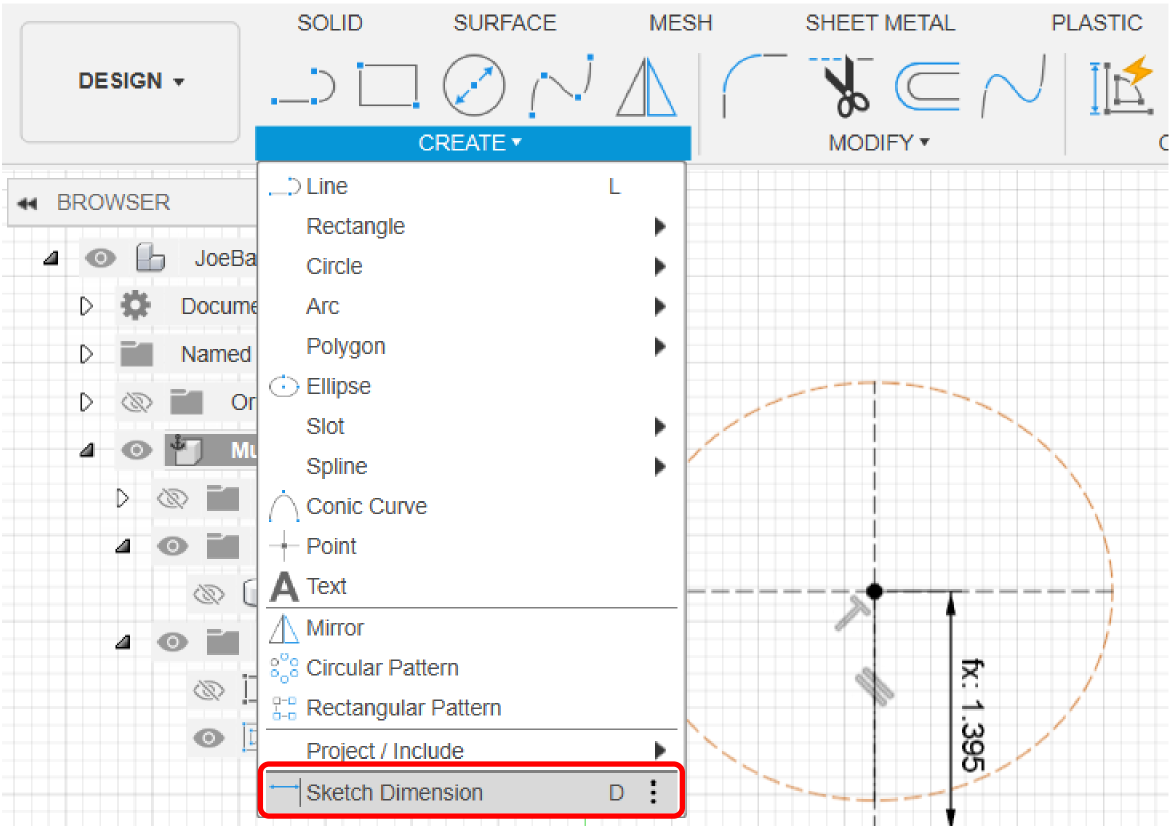
- click at the **top of the line** to start the ellipse and **extend it out to the left** by any amount and click



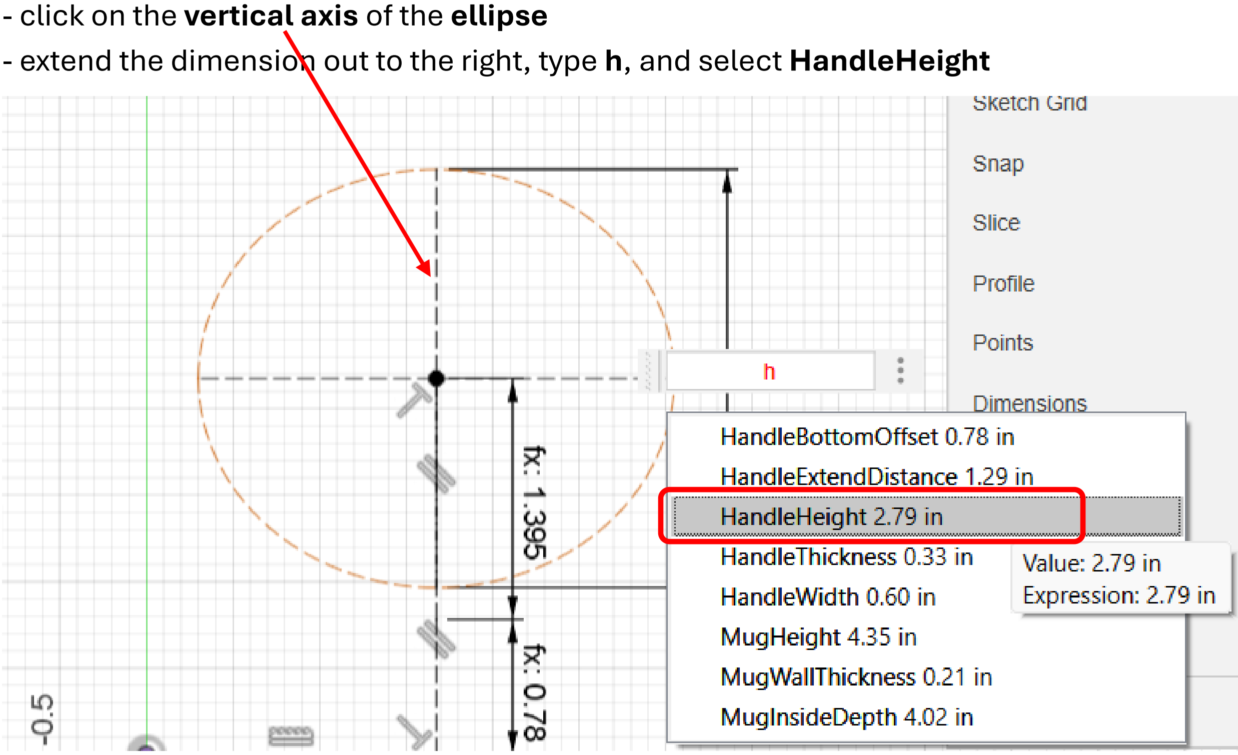
- **extend the ellipspe up** by any amount and click, which should result in an ellipse comprised of a dashed orange line. It is difficult to select parameter values when creating an ellipse. This will be done later.



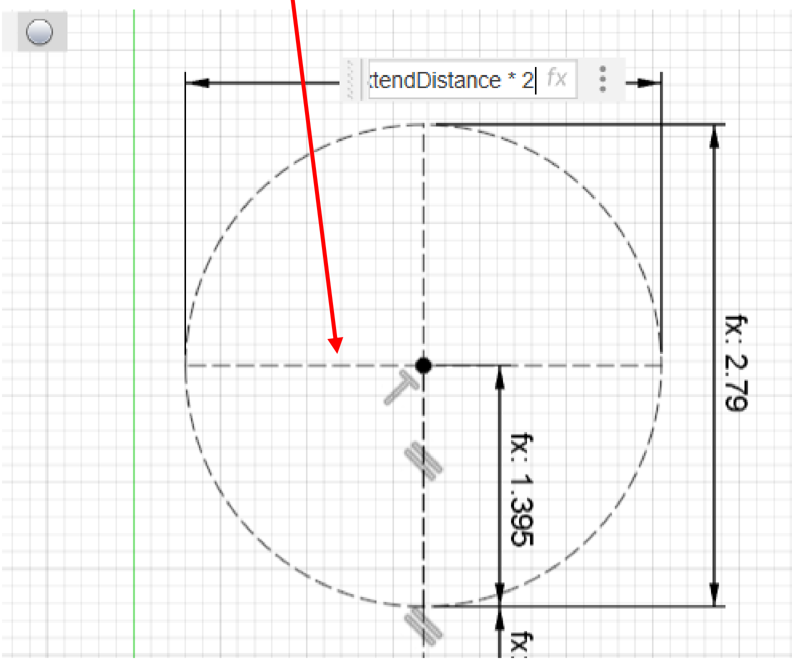
-from the **CREATE** menu select **Sketch Dimension**



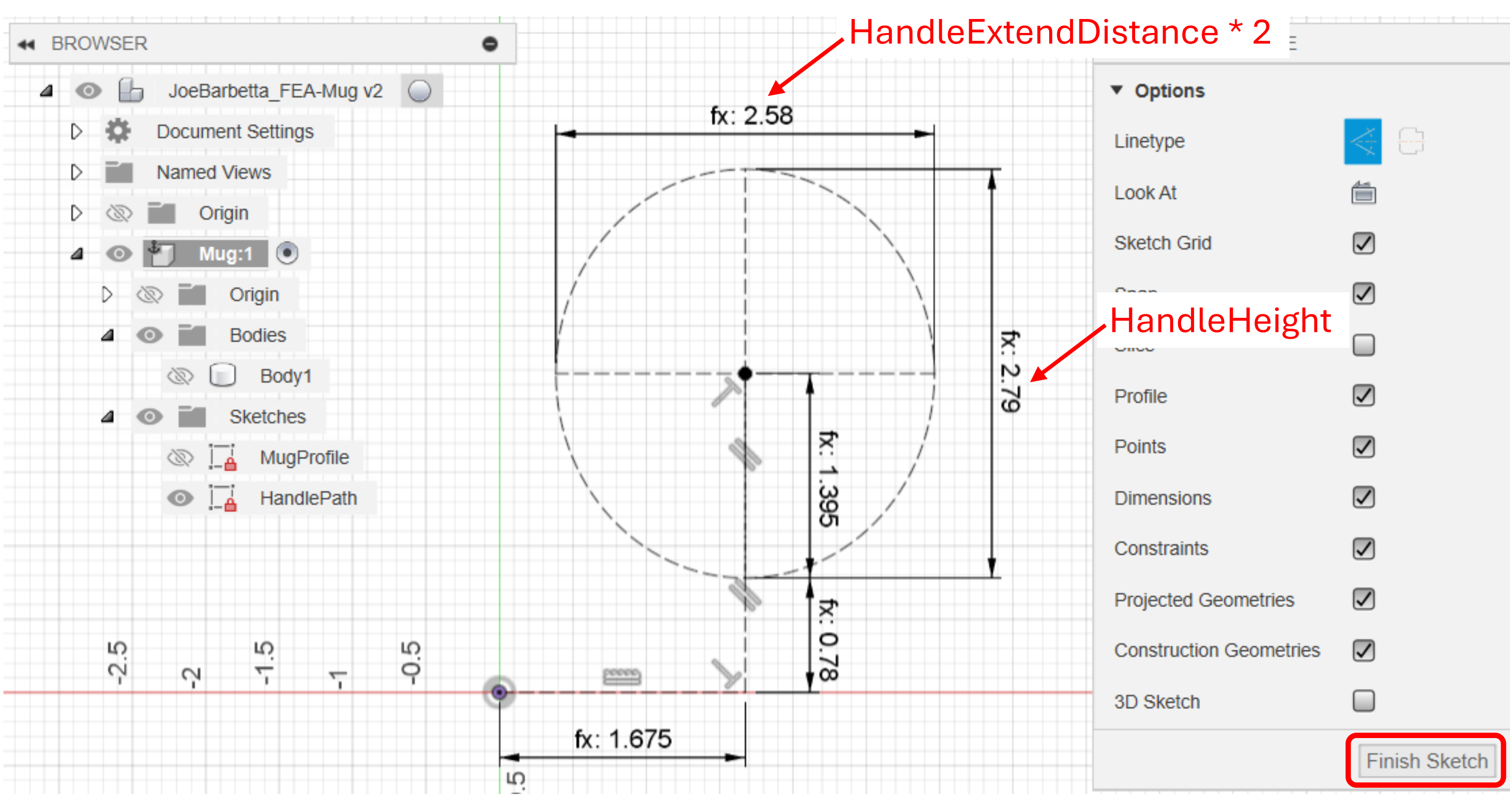
- click on the **vertical axis** of the **ellipse**
- extend the dimension out to the right, type **h**, and select **HandleHeight**



- click on the **horizontal axis** of the **ellipse**
- extend the dimension upward, type **h**, select **HandleExtendDistance**, and type *** 2** after it

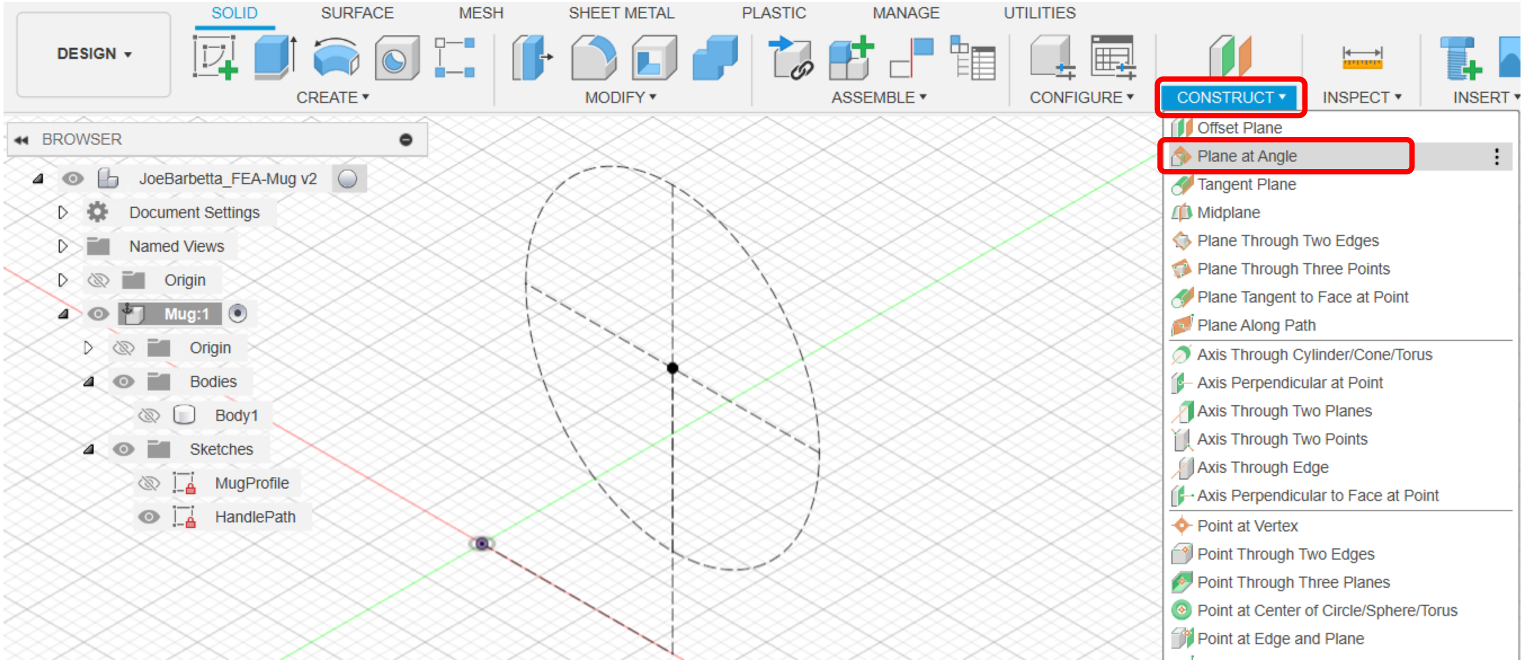


This should be the result of this sketch.
- click **Finish Sketch**

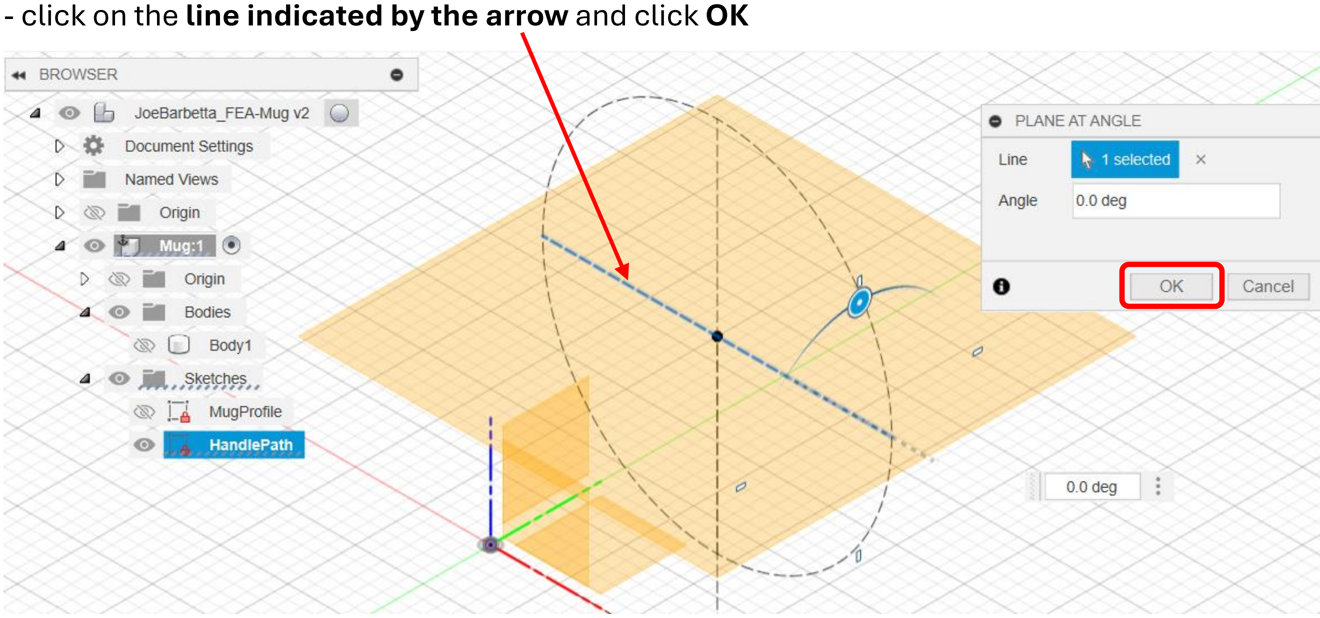


Creating the Handle Profile Sketch

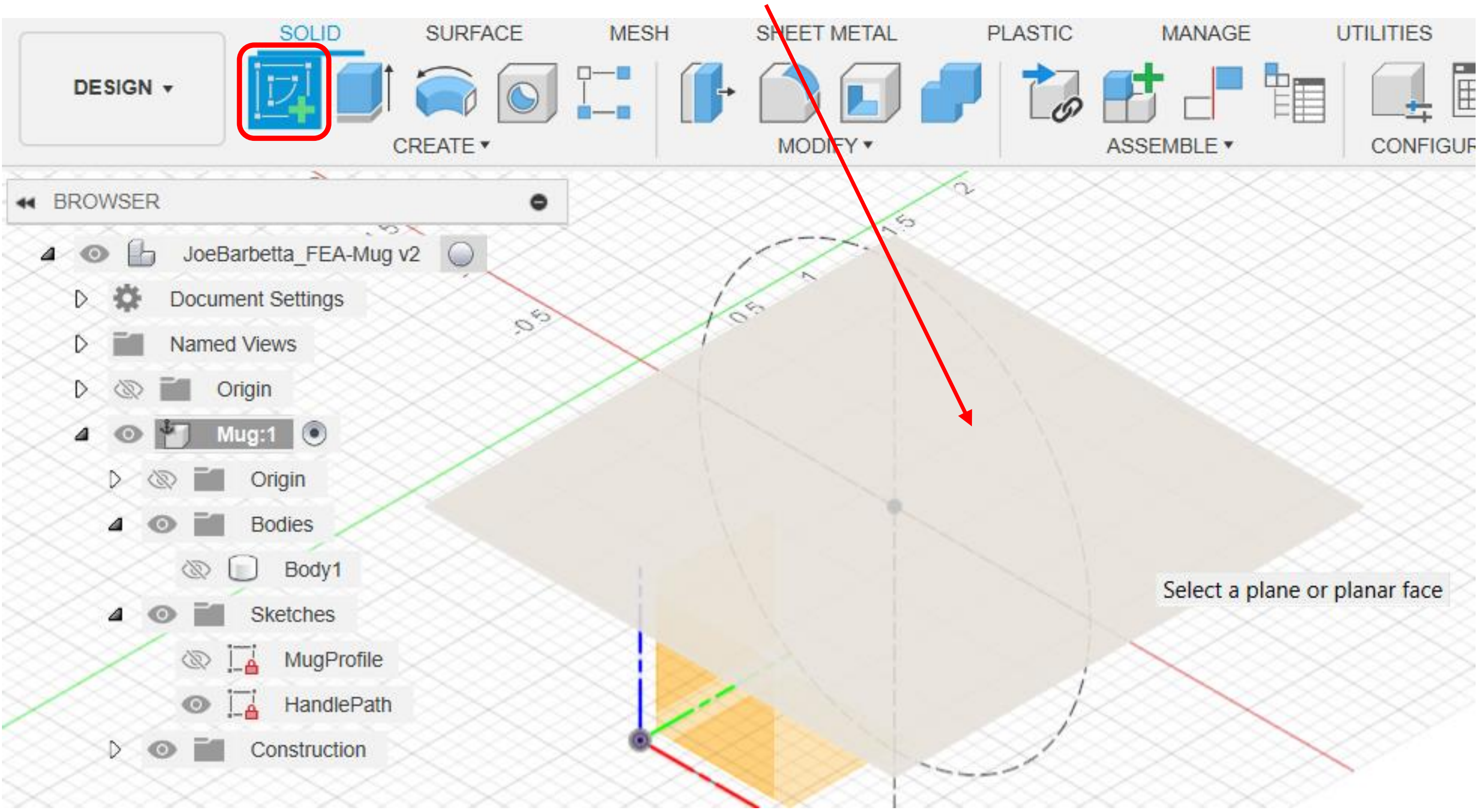
- click on the **Home** icon at the **View Cube** to achieve the view similar to that below
- from the **CONSTRUCT** menu near the top right select **Plane at Angle**



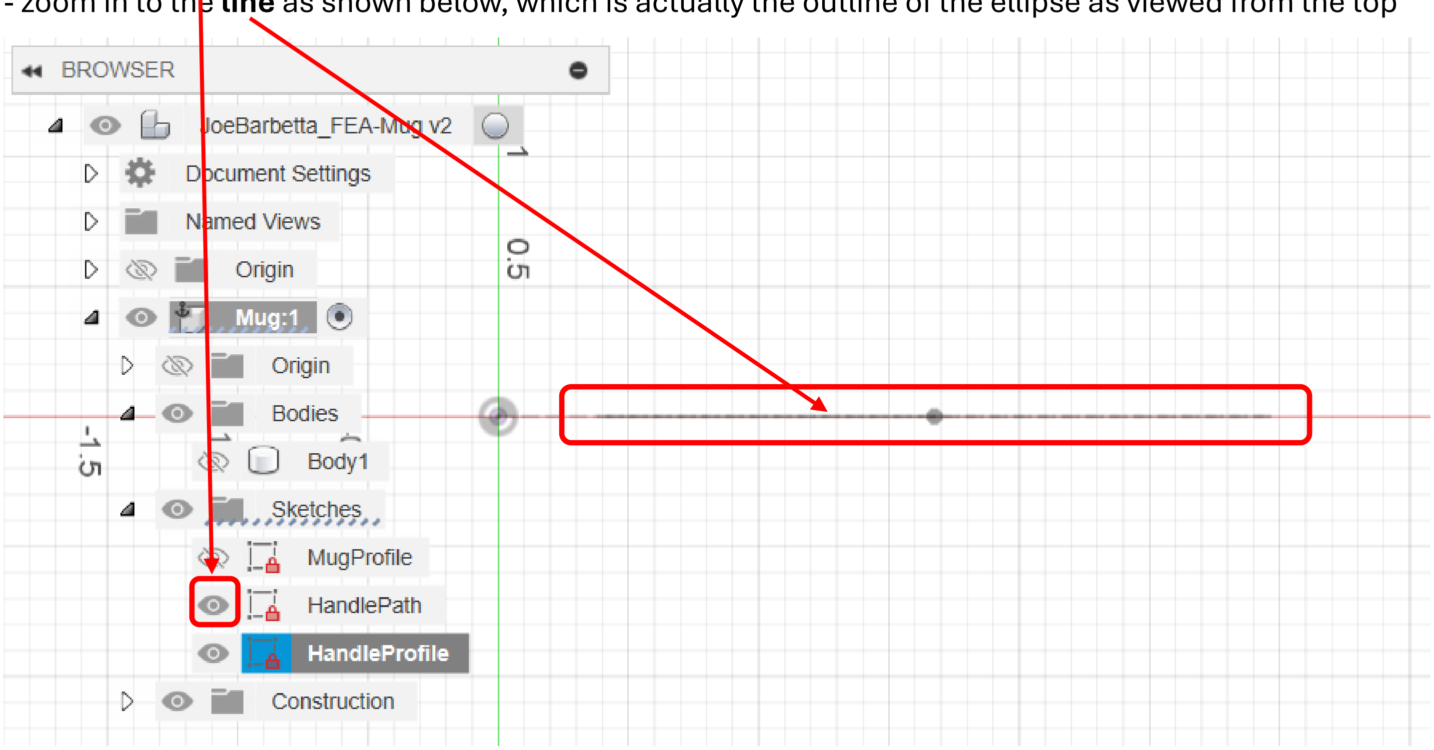
- click on the **line indicated by the arrow** and click **OK**



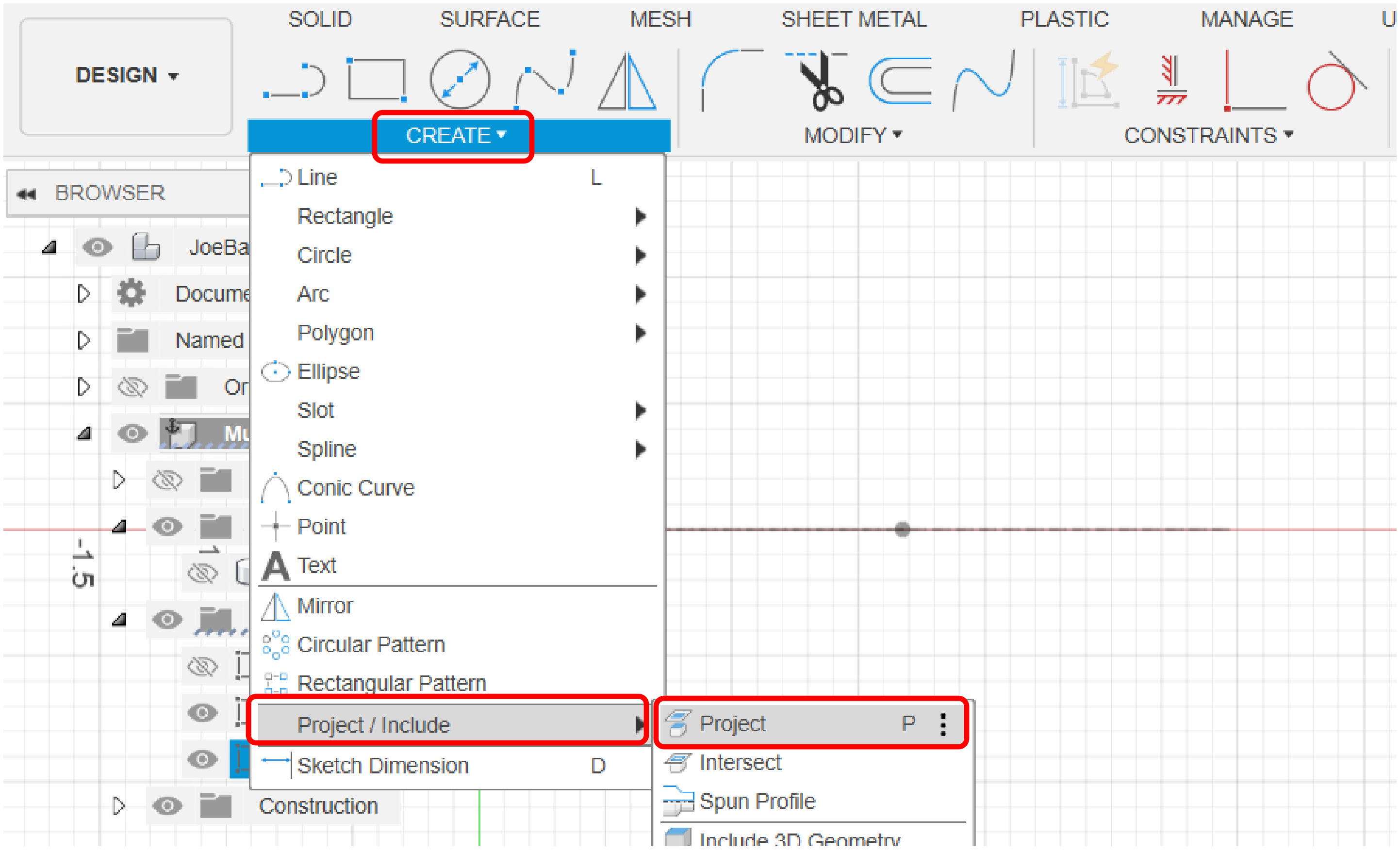
- select **Create Sketch** and click on the orange **Construction Plane** that was just created



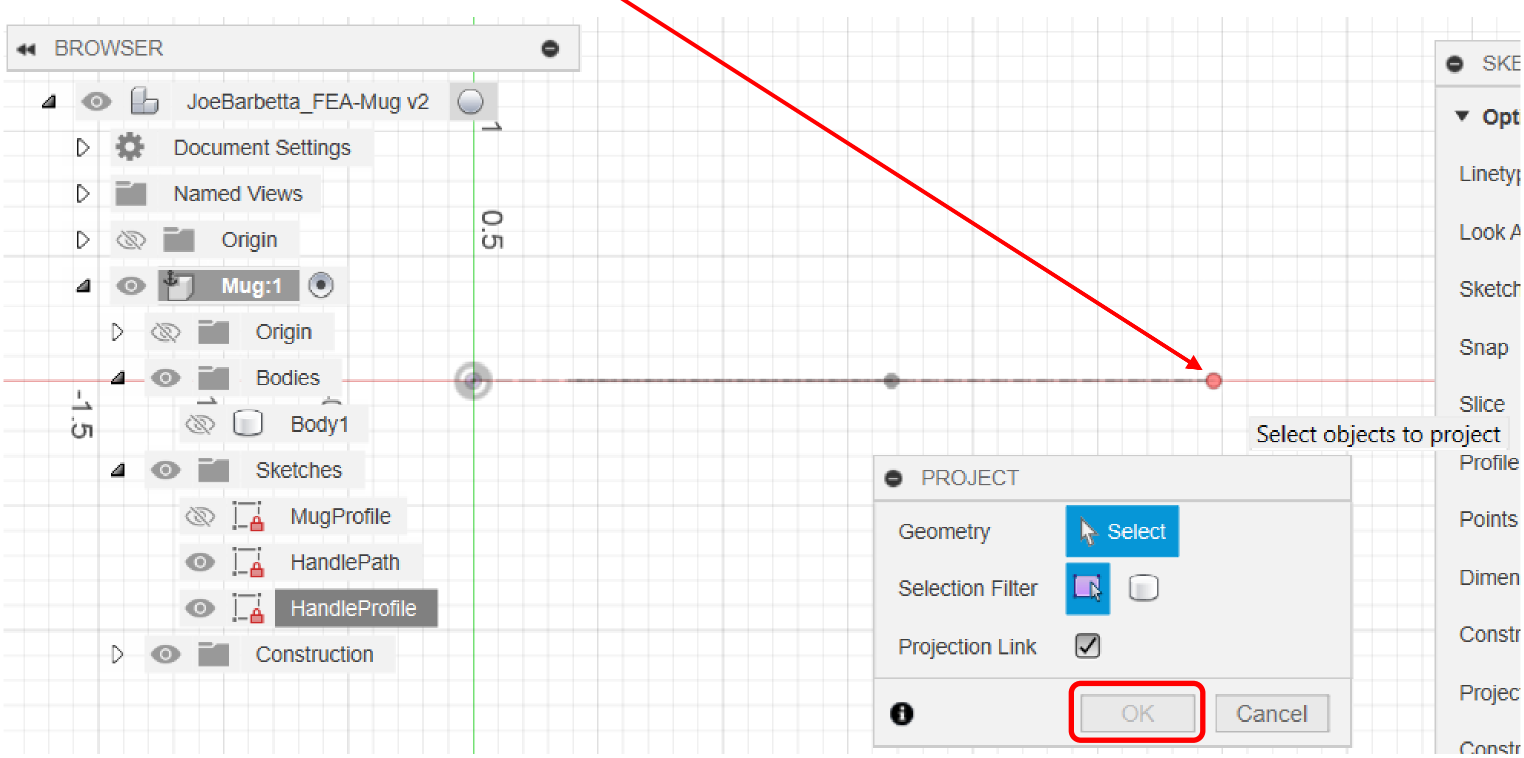
- ensure the **eye** icon is enabled for the **HandlePath** sketch
- zoom in to the **line** as shown below, which is actually the outline of the ellipse as viewed from the top



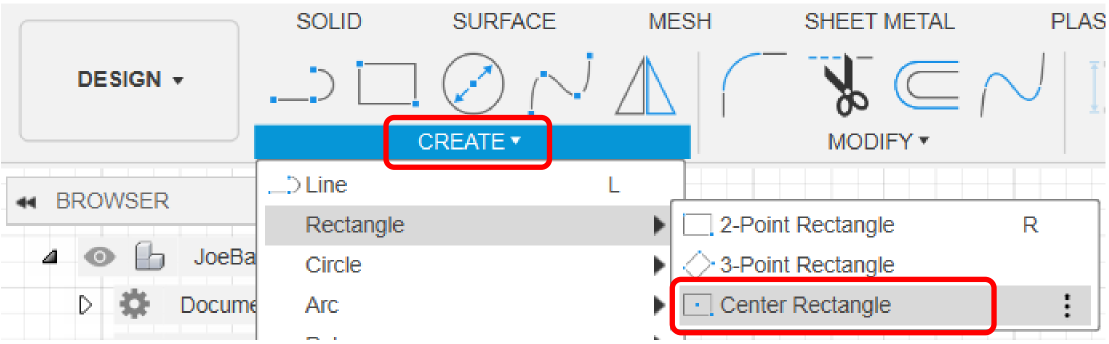
- from the **CREATE** menu select **Project / Include** and **Project**



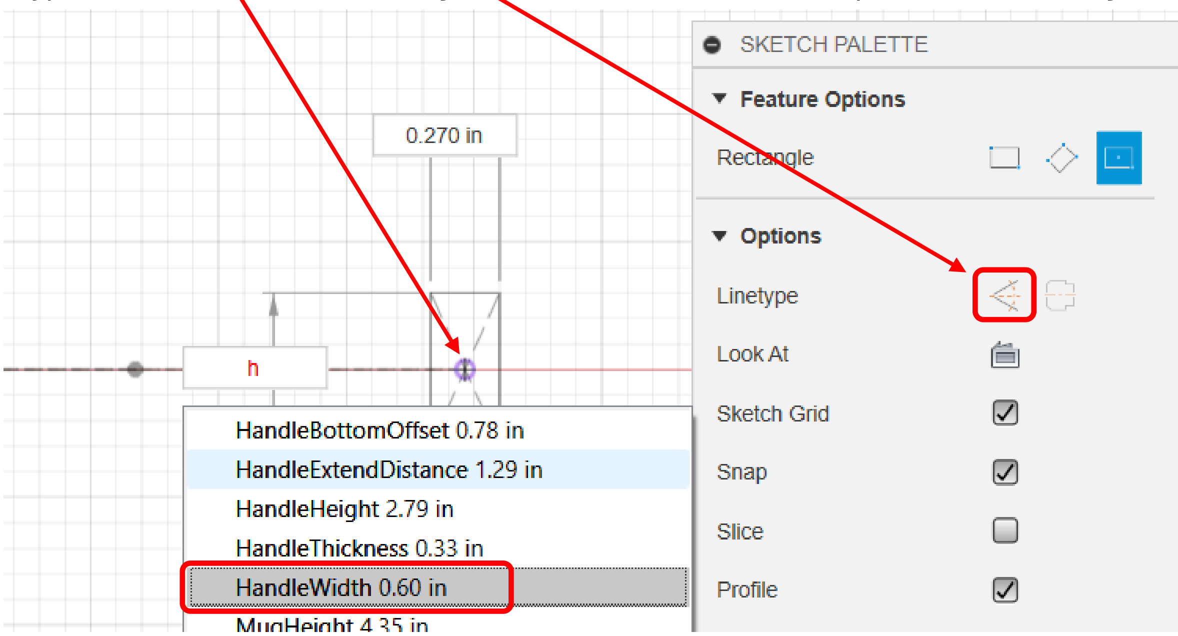
- move the mouse over the **right endpoint of the line**, which should result in a red circle
- click on the **point** and click **OK**



- from the **CREATE** menu select **Center Rectangle**

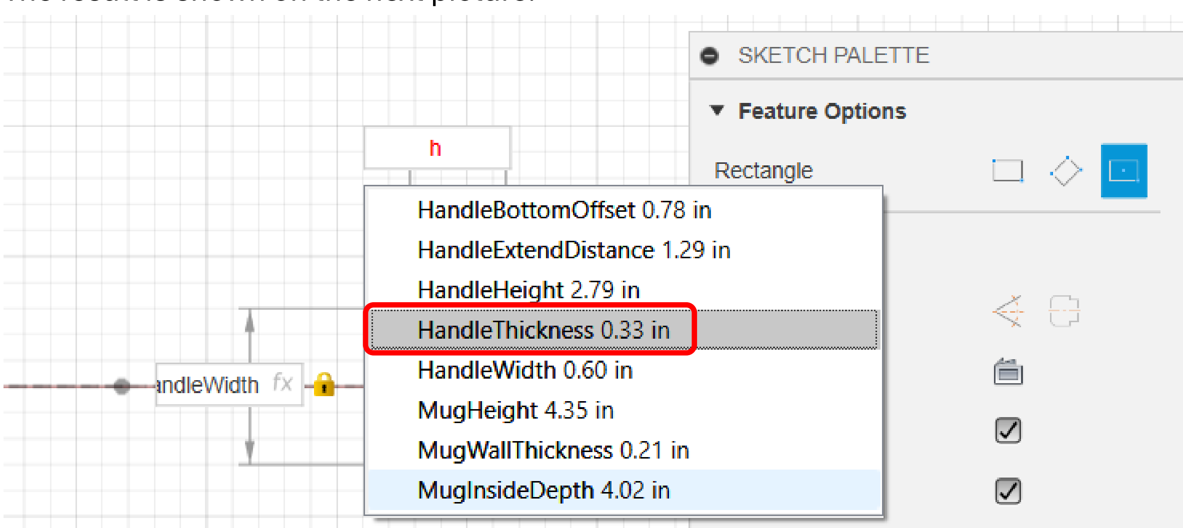


- ensure that the **Construction** icon is not highlighted
- click on the **point** that was just created and **extend the rectangle down and to the right**
- type **h**, use the **down arrow key** to select **HandleWidth**, and press the **Enter key**

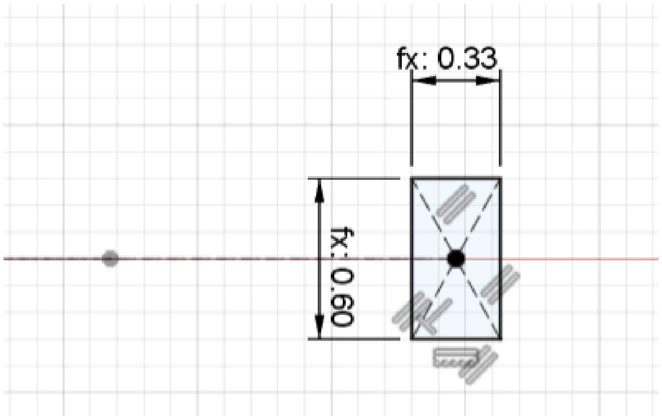


- press the **Tab key** to switch to the top dimension
- type **h**, use the **down arrow key** to select **HandleThickness**, and press the **Enter key twice**

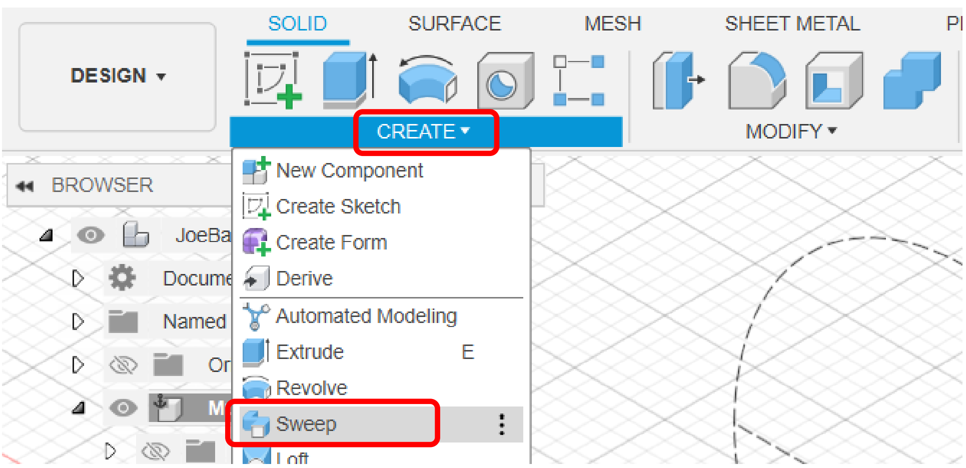
The result is shown on the next picture.



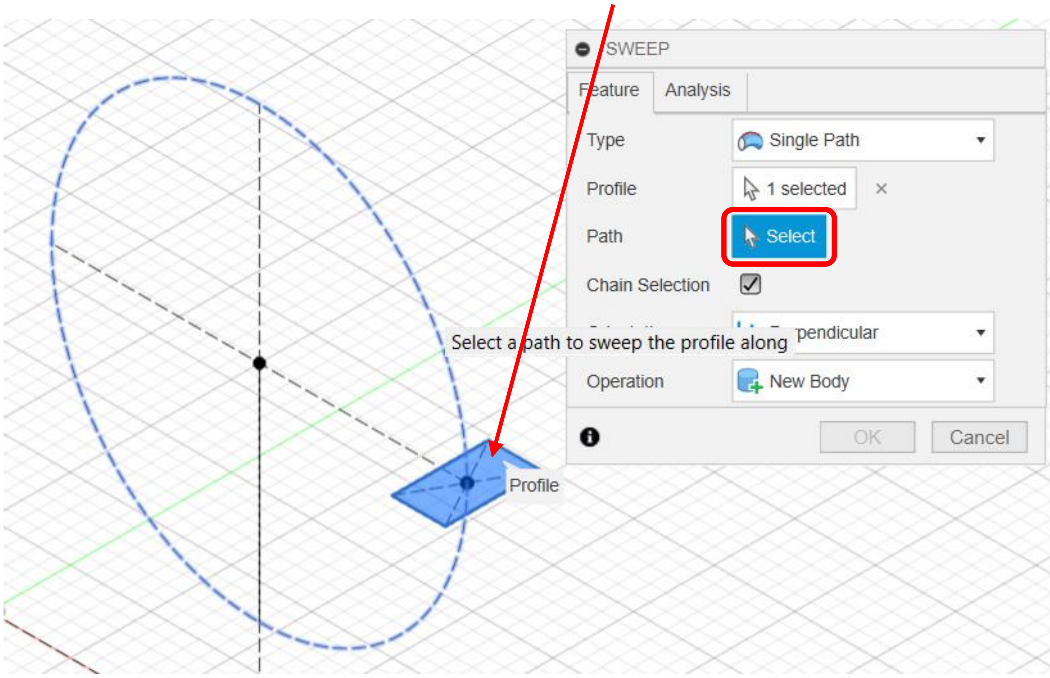
- click **Finish Sketch**



- click on the **Home** icon at the **View Cube**
- from the **CREATE** menu select **Sweep**

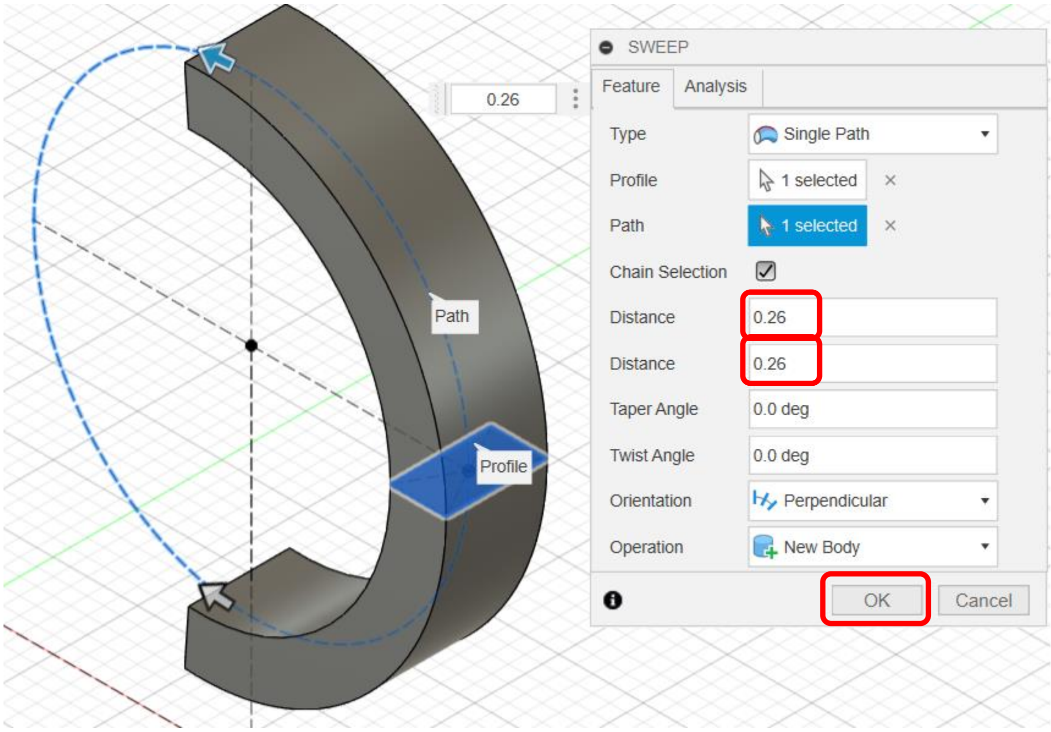


- click on the **rectangle**
- click on **Select** for **Path** and click on the **ellipse**, which should result in a full gray ring

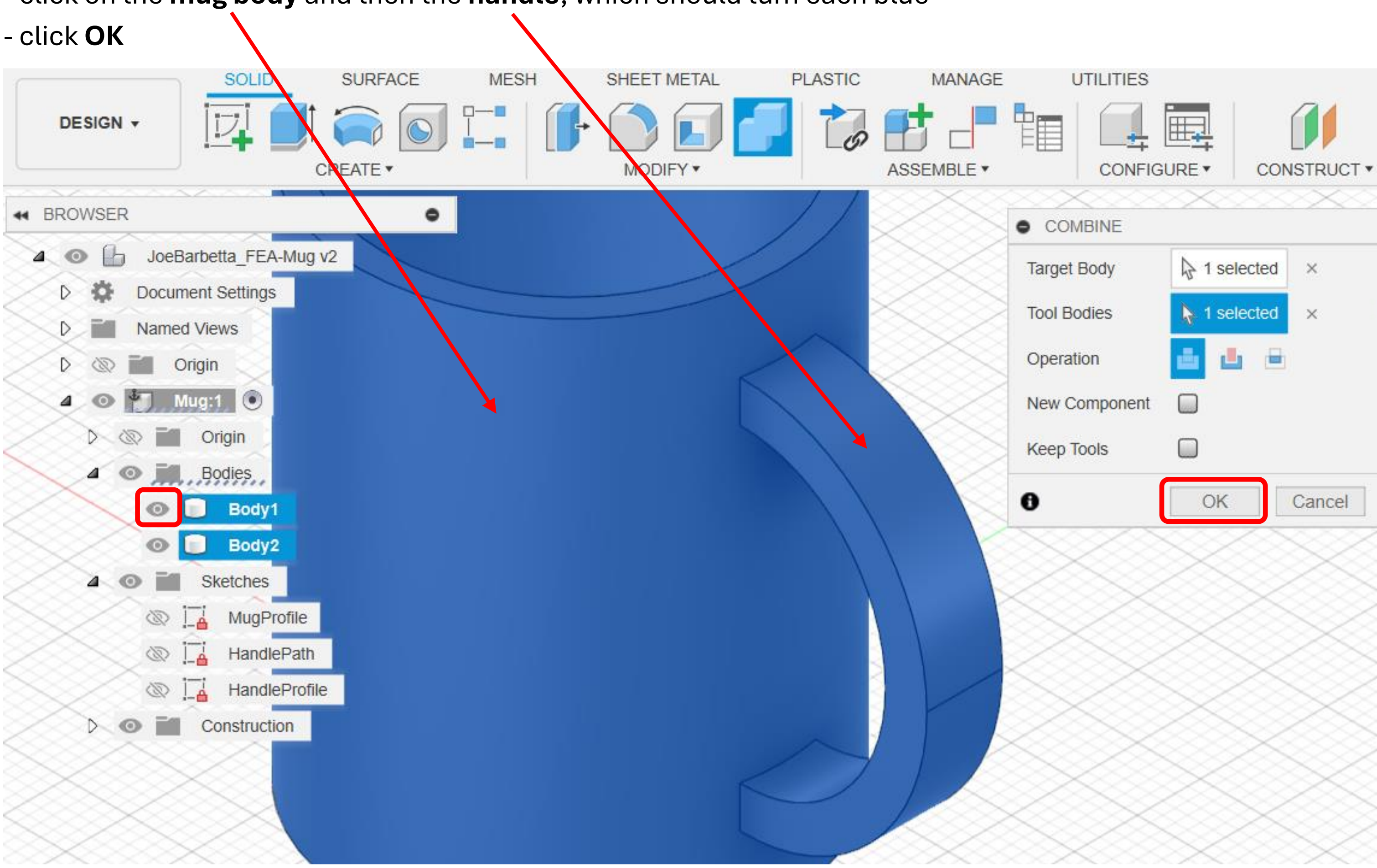


We don't want the handle to also be in the mug, so we will limit its distance.

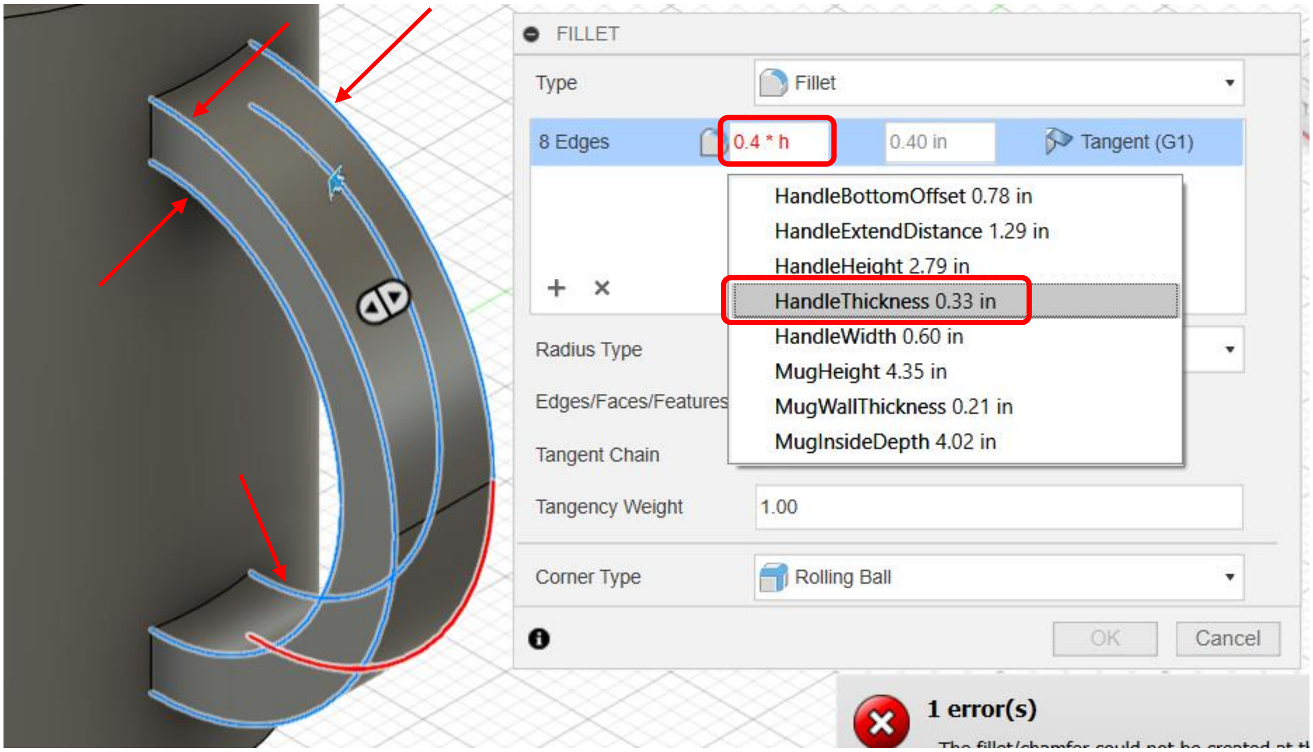
- set both **Distance** values to **0.26** and click **OK**



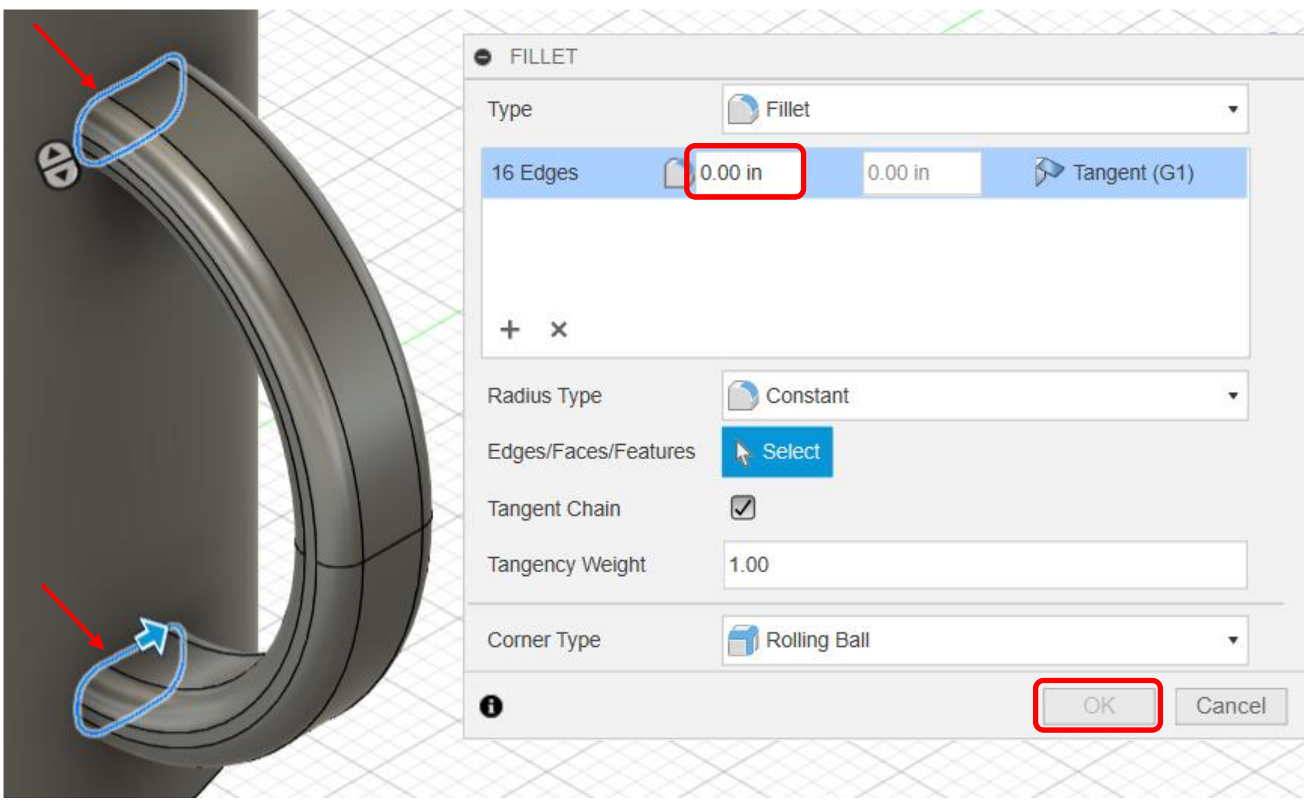
- click on the **eye** icon for the **1st Body** to make the mug body visible
- select the **Combine** tool. If it is not visible find it in the MODIFY menu.
- click on the **mug body** and then the **handle**, which should turn each blue
- click **OK**



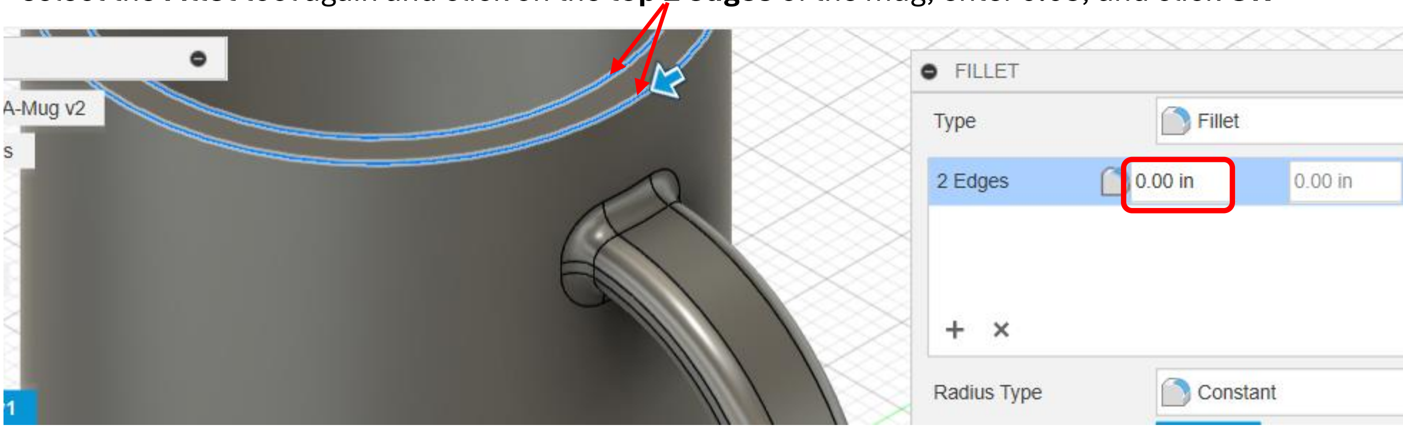
- from the **MODIFY** menu select **Fillet**
- click on the **4 edges** as indicated by the arrows
- in the value box type **0.4 * h** and then select **HandleThickness**. An error may show at first, until the full expression is entered.
- click **OK**



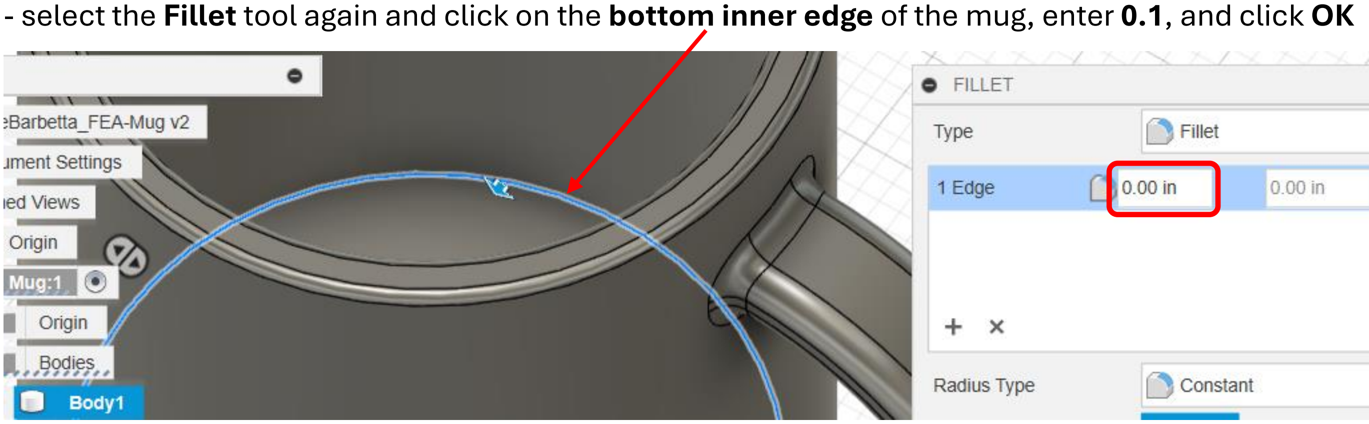
- select the **Fillet** tool again and click on the **interface of the top and bottom handle ends and the mug**
- enter **0.125** and click **OK**



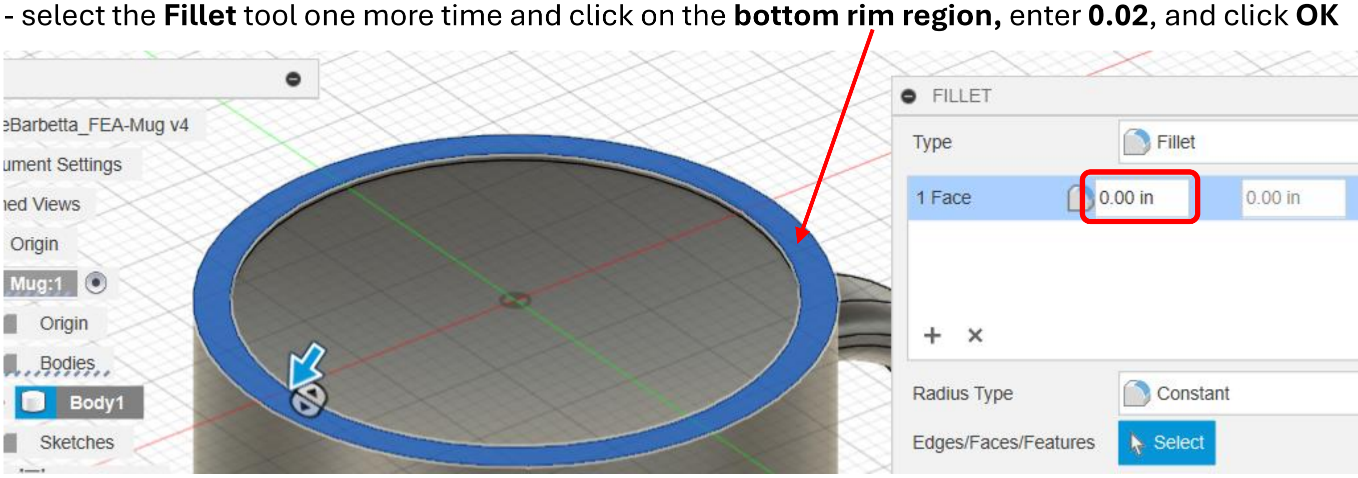
- select the **Fillet** tool again and click on the **top 2 edges** of the mug, enter **0.05**, and click **OK**



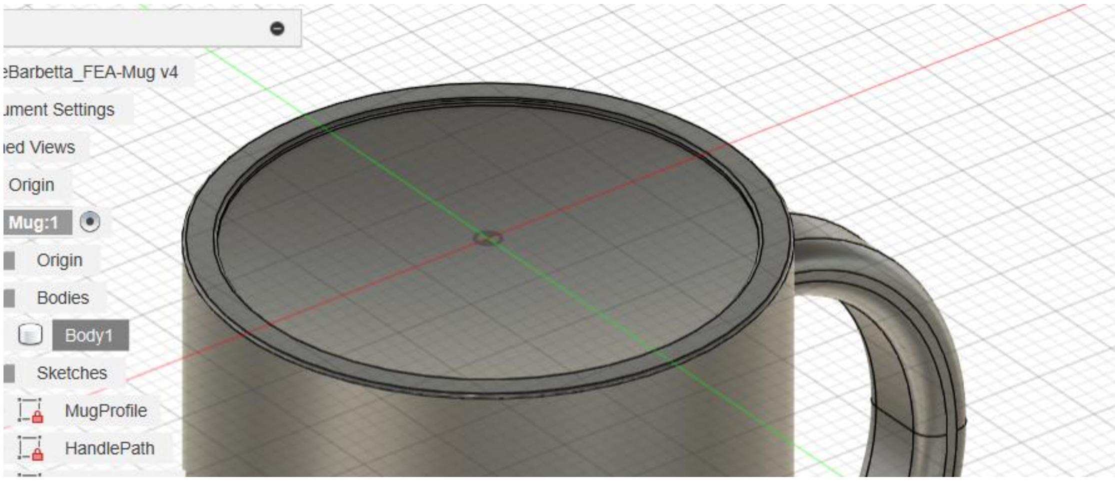
- adjust the view to access the inside of the mug
- select the **Fillet** tool again and click on the **bottom inner edge** of the mug, enter **0.1**, and click **OK**



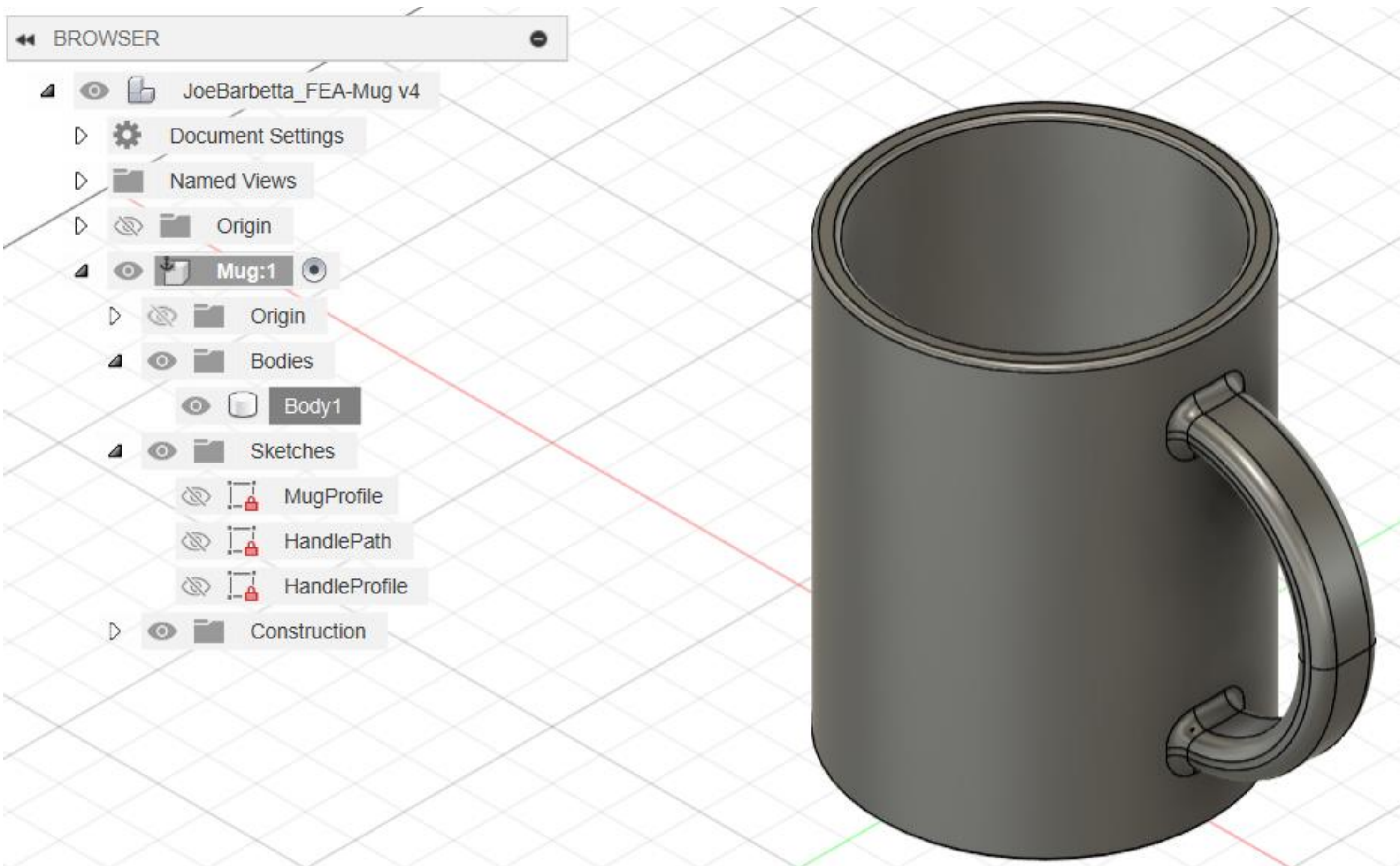
- adjust the view to access the **bottom of the mug**
- select the **Fillet** tool one more time and click on the **bottom rim region,** enter **0.02**, and click **OK**



This is the result of the bottom fillets.

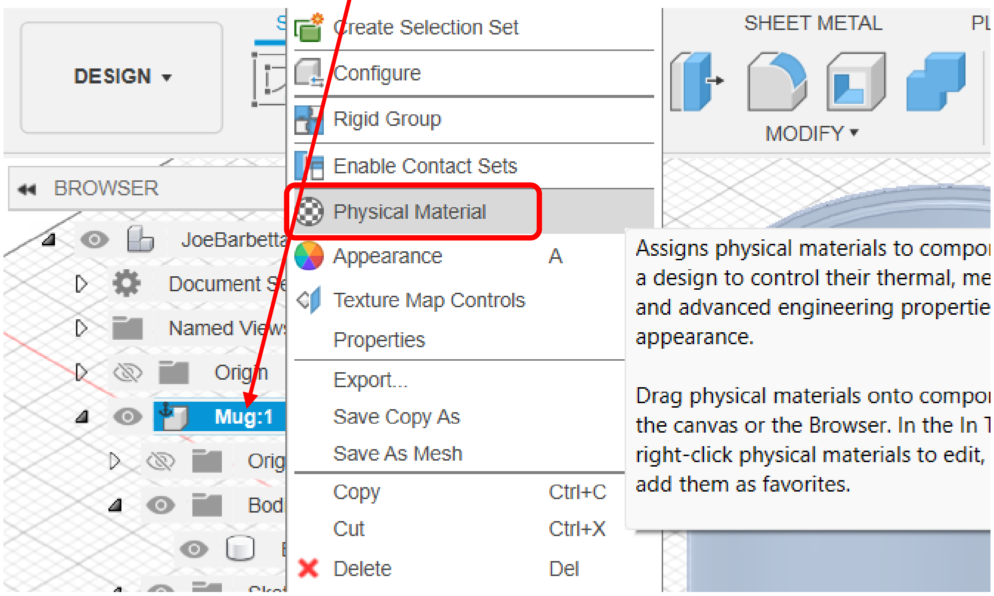


- return to the Home view and admire your mug

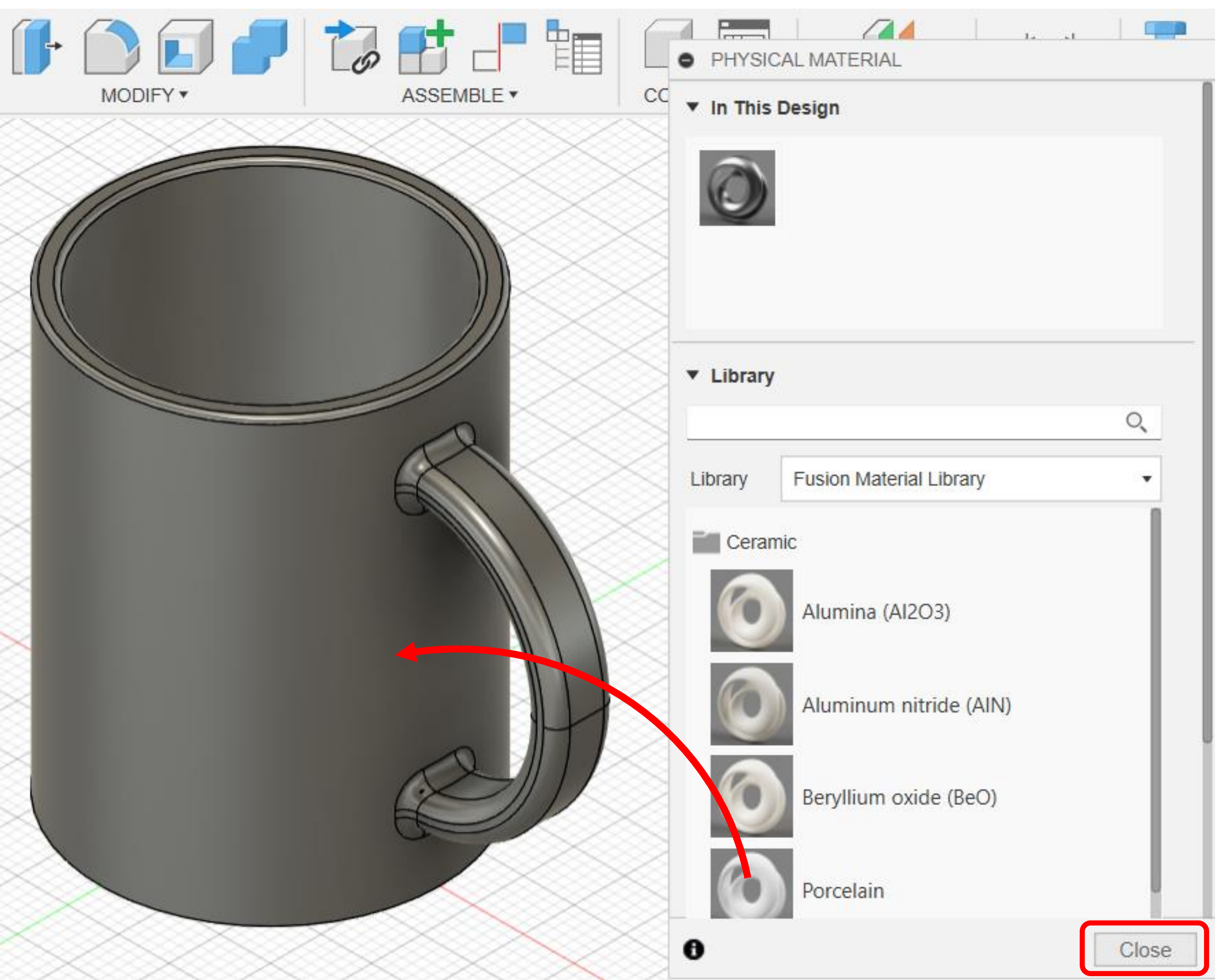


Setting the material

- right-click on the **Component name** and select **Physical Material**

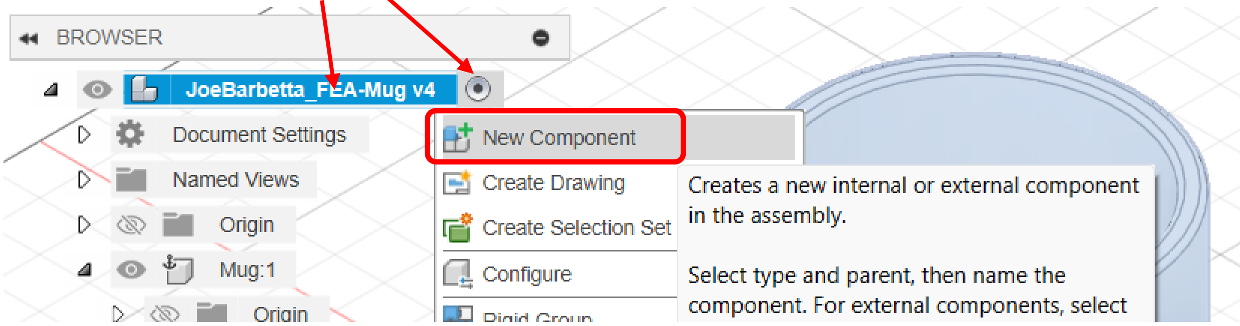


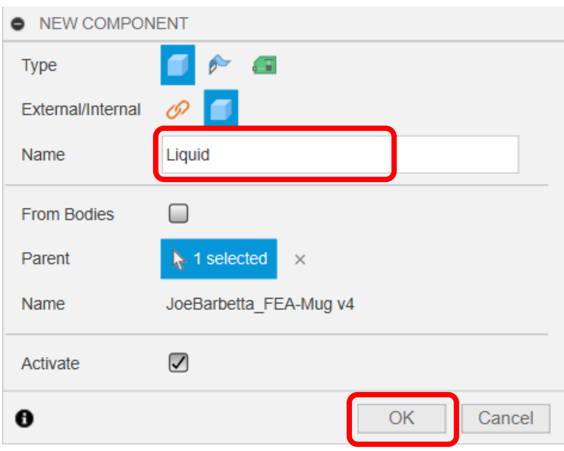
- click on the **Ceramic folder** to open it and scroll down to **Porcelain**
- drag the **Porcelain icon** unto the mug and click **Close**



Adding Water

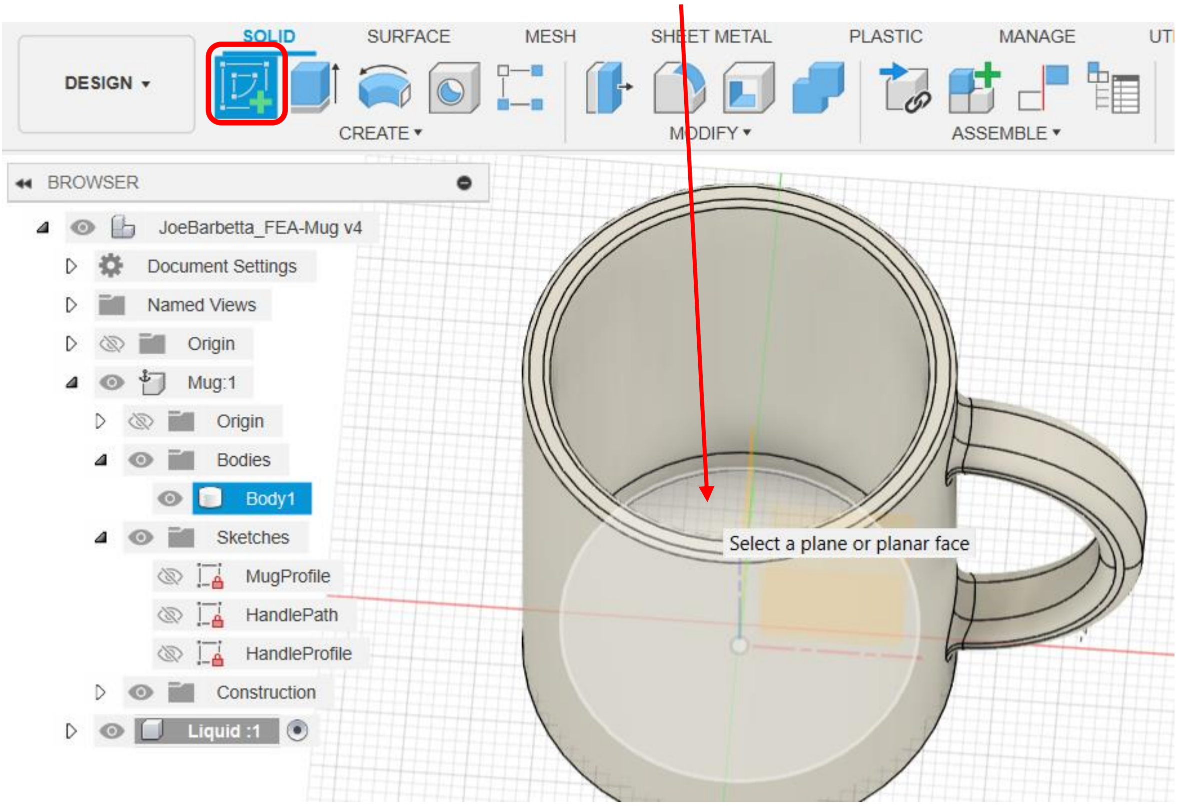
- click on the **circle for the design** to activate the entire design
- right-click on the **project name** and select **New Component**



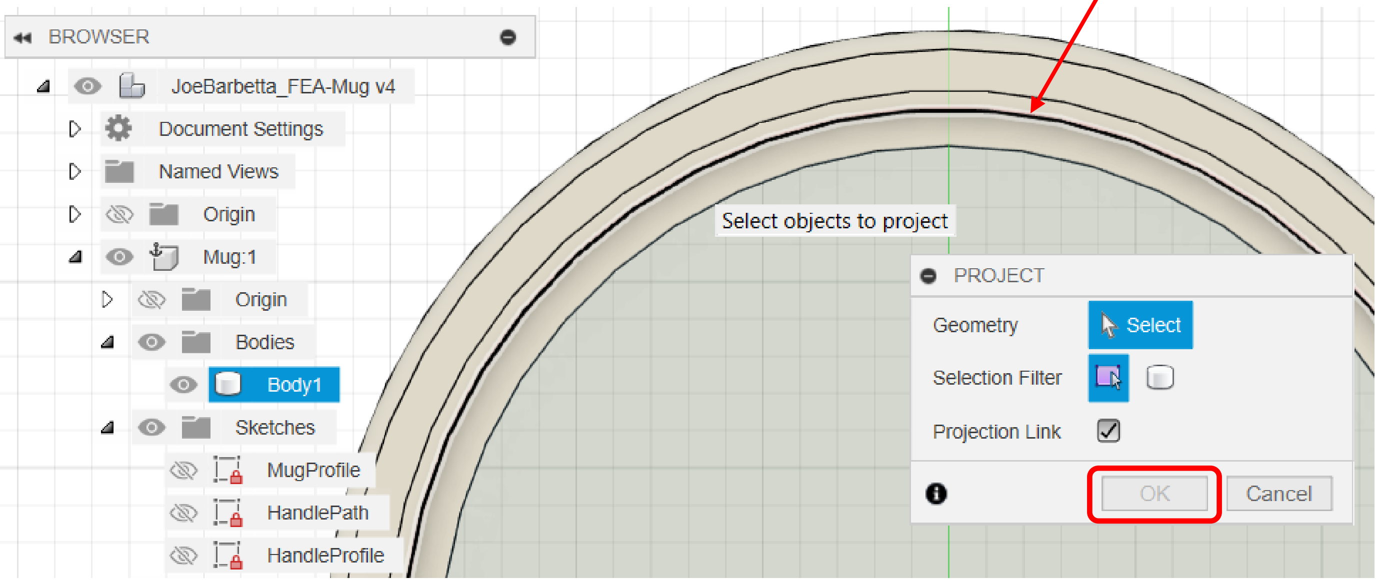


- change the Name to **Liquid** and click **OK**

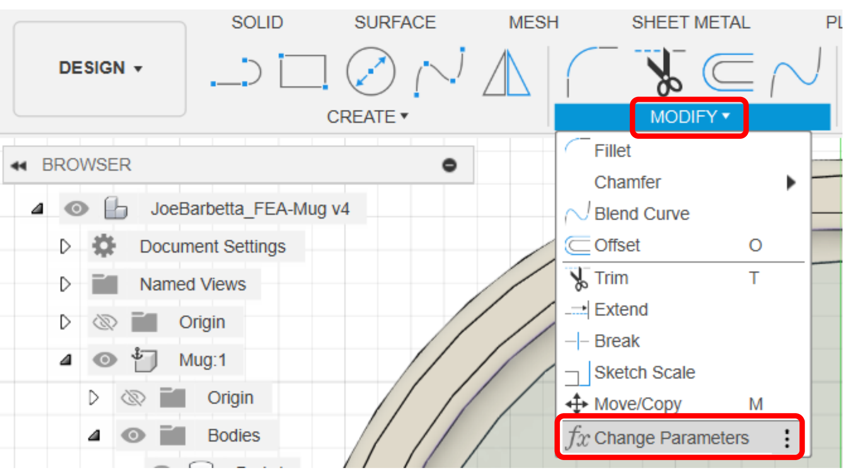
- adjust the view to access the inside of the mug
- select **Create Sketch** and click on the **inner bottom surface**



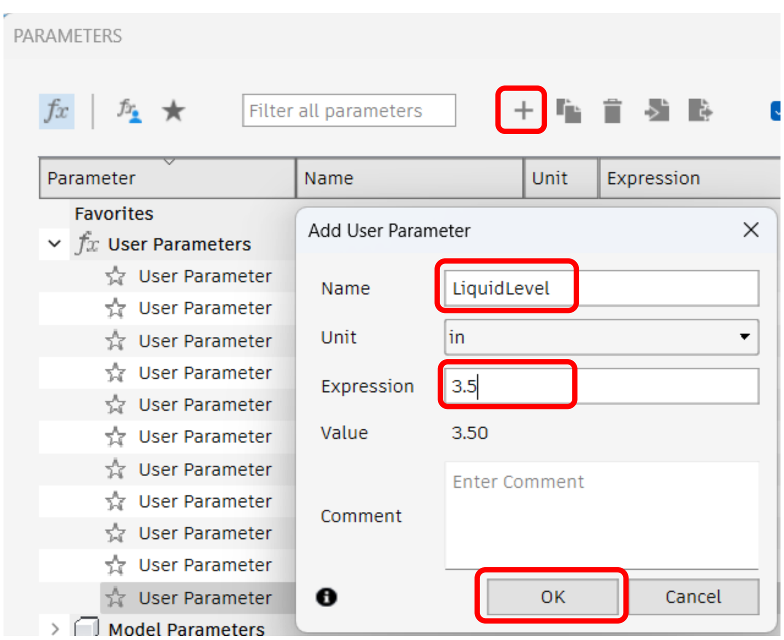
- zoom in to the Sketch as shown below
- from the **CREATE** menu select **Project / Include** and **Project** and click on the **circle edge** indicated
- click **OK**



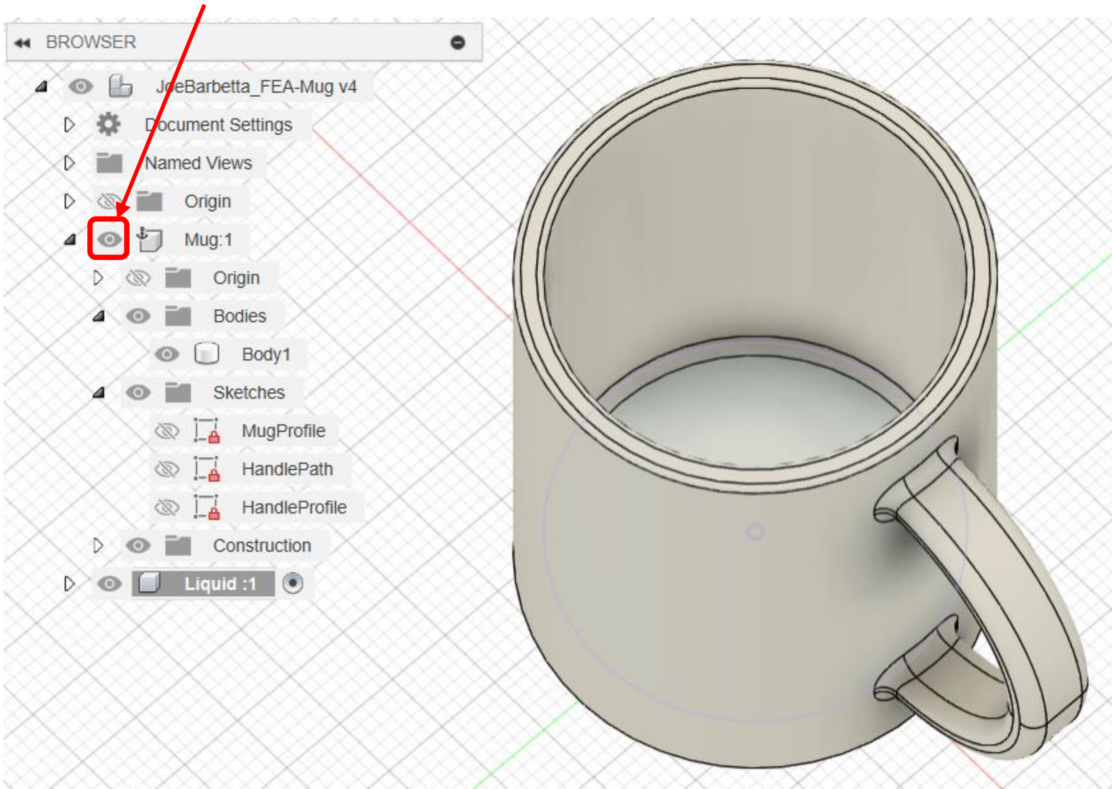
- from the **MODIFY** menu select **Change Parameters**



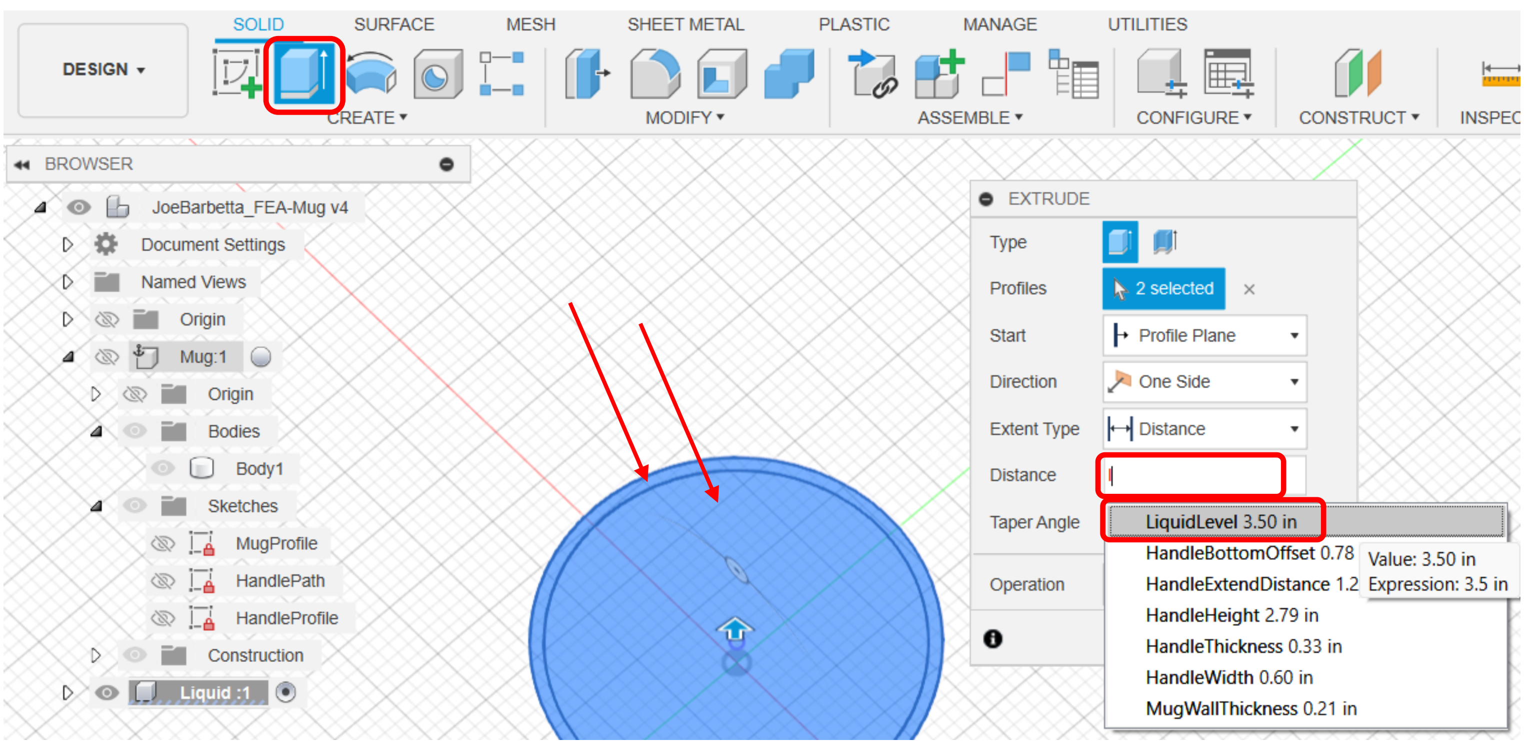
- click the **+** icon
- set the **Name** to **LiquidLevel** and the **Expression** to **3.5**
- click **OK**
- click **OK** at the bottom of the Parameters window



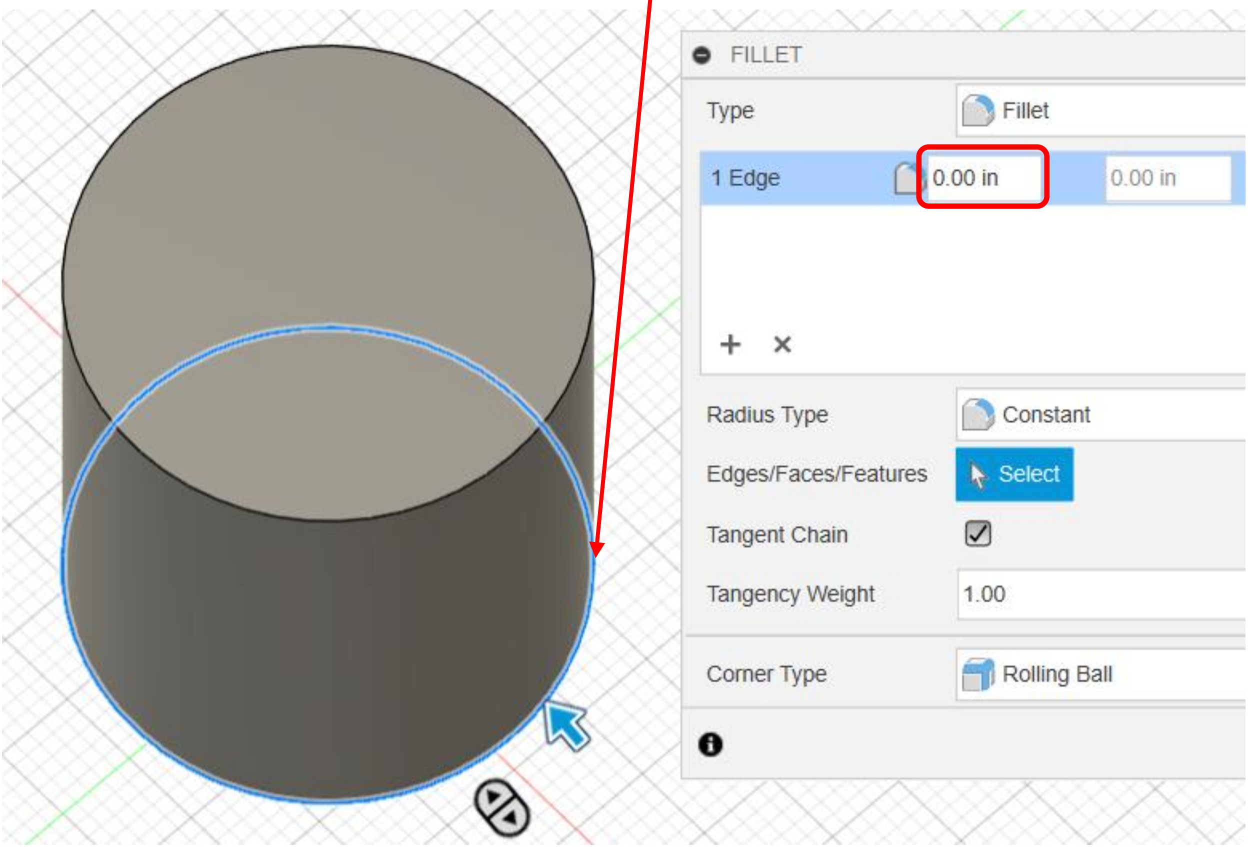
- adjust the view to access the **inner bottom surface**
- click on the **eye** icon for the Mug to hide it



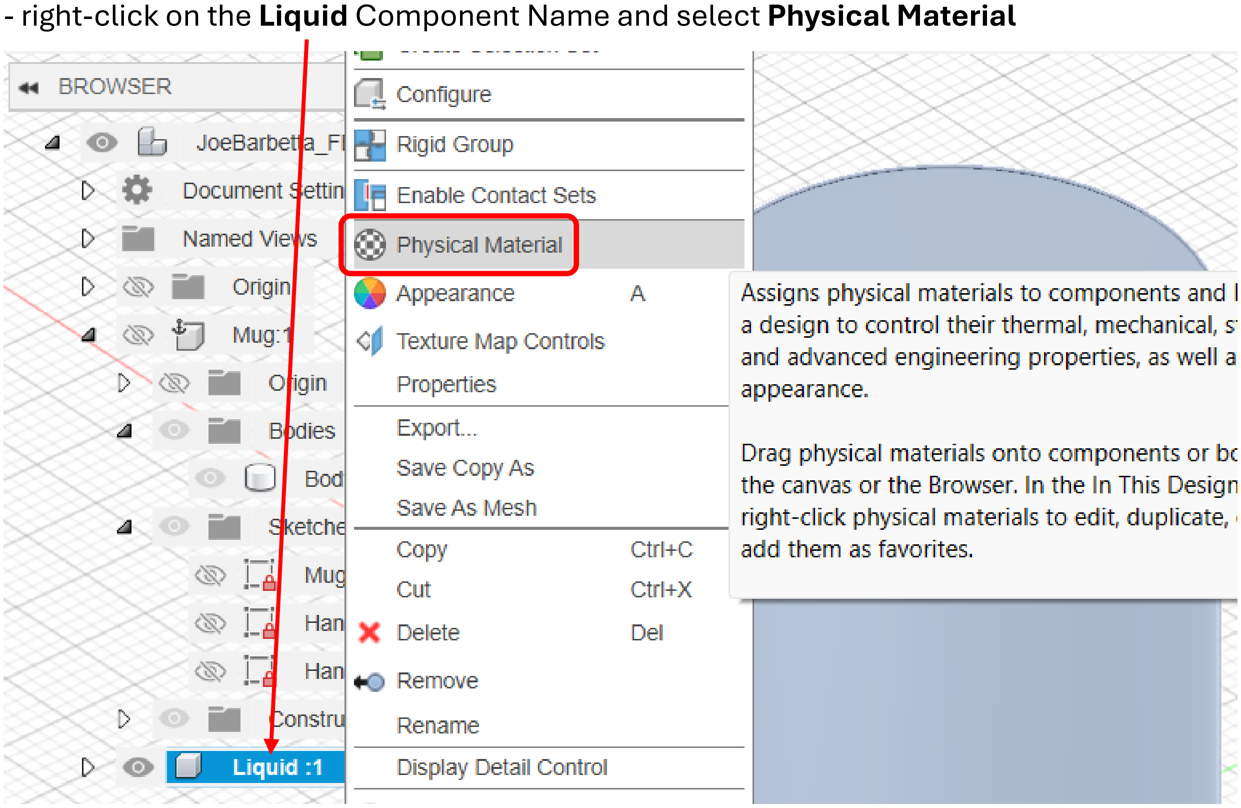
- select the **Extrude** tool. If it is not visible find it in the **CREATE** menu.
- click on the **inner circle** and the **outer ring** to highlight them blue
- in the **Distance** box type **l** and seleect **LiquidLevel** and click **OK**



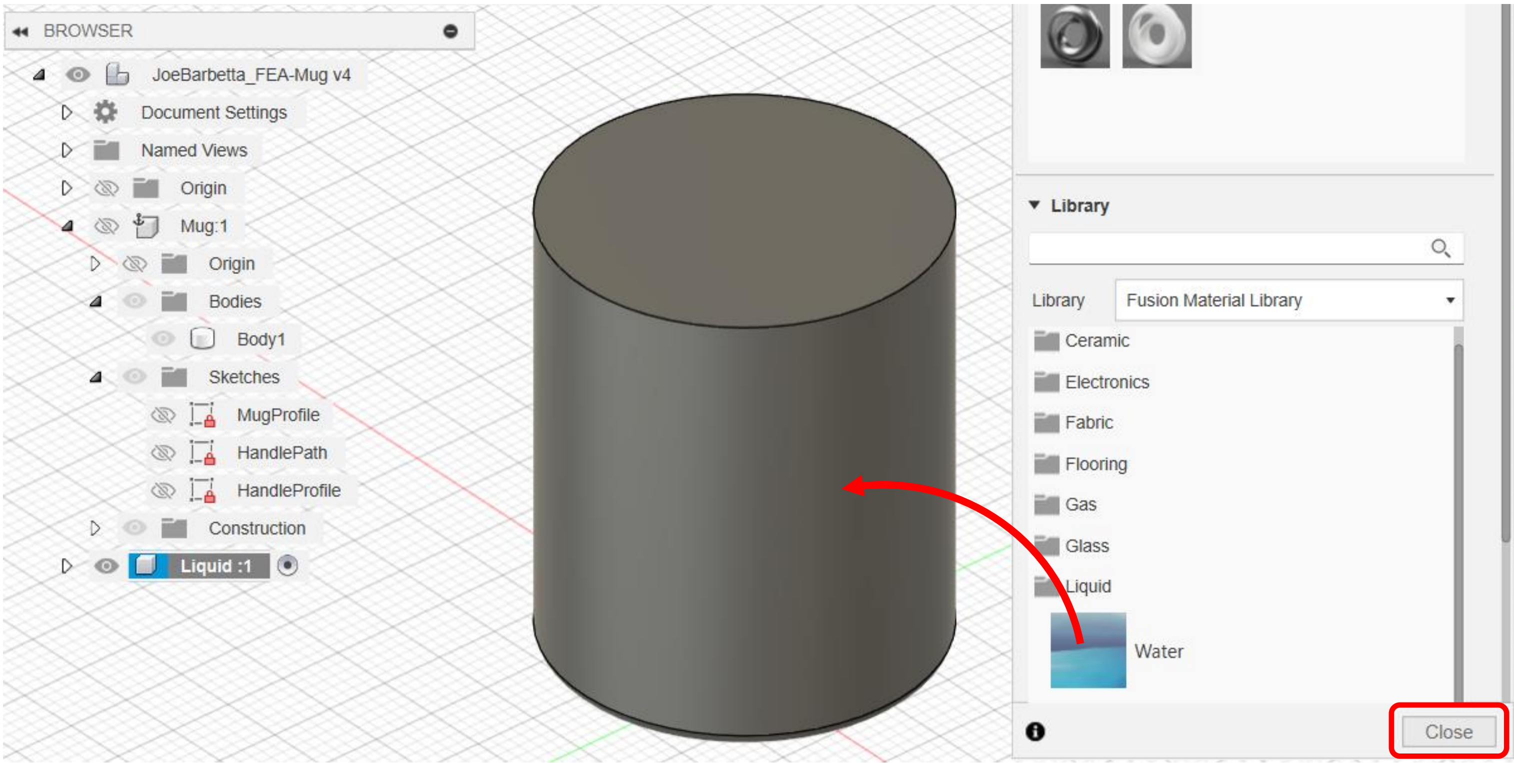
- select the **Fillet** tool and select the **bottom edge**
- enter **0.1** and click **OK**



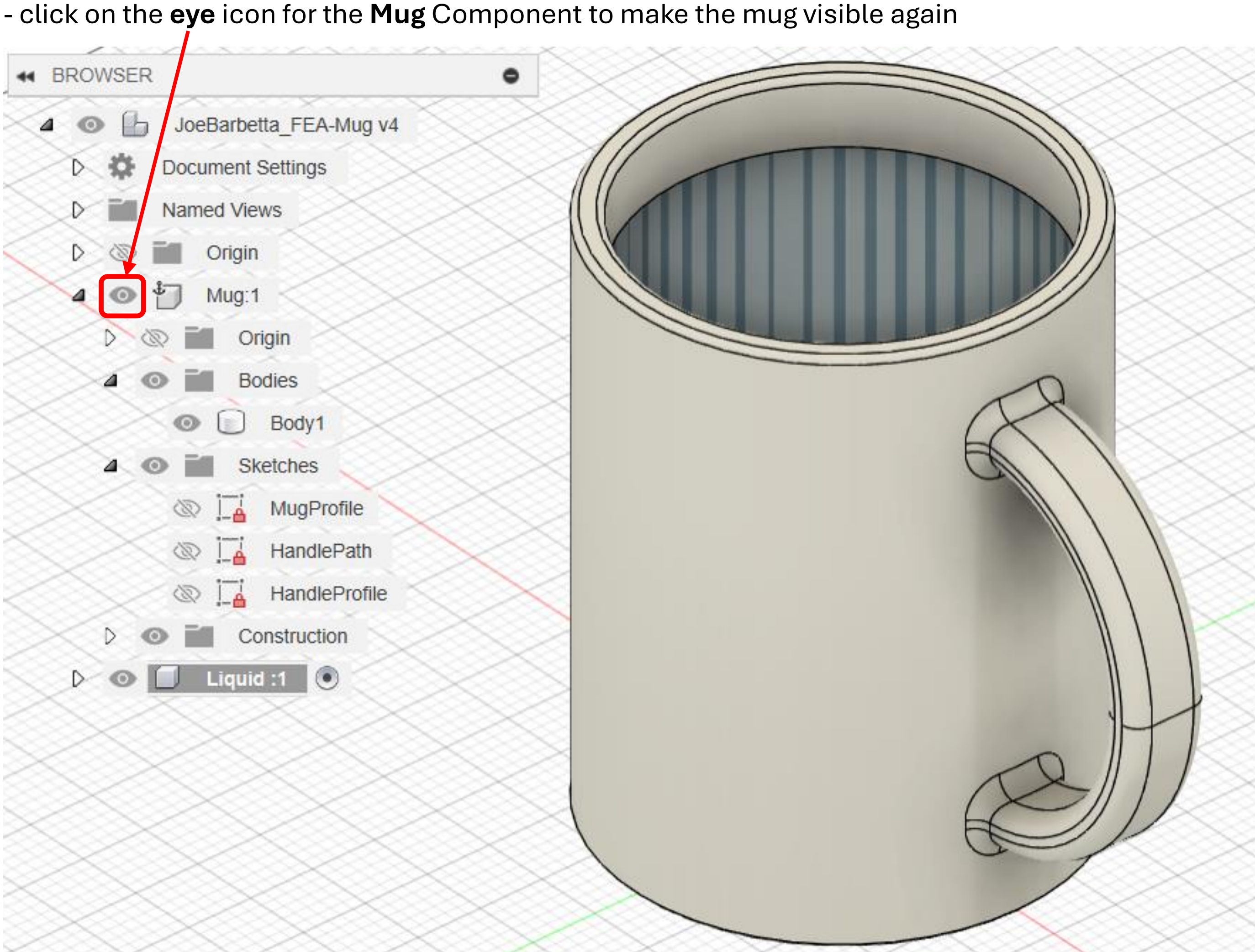
- right-click on the **Liquid** Component Name and select **Physical Material**



- scroll down and click on the **Liquid** folder and drag the **Water** icon onto the cylinder and click Close

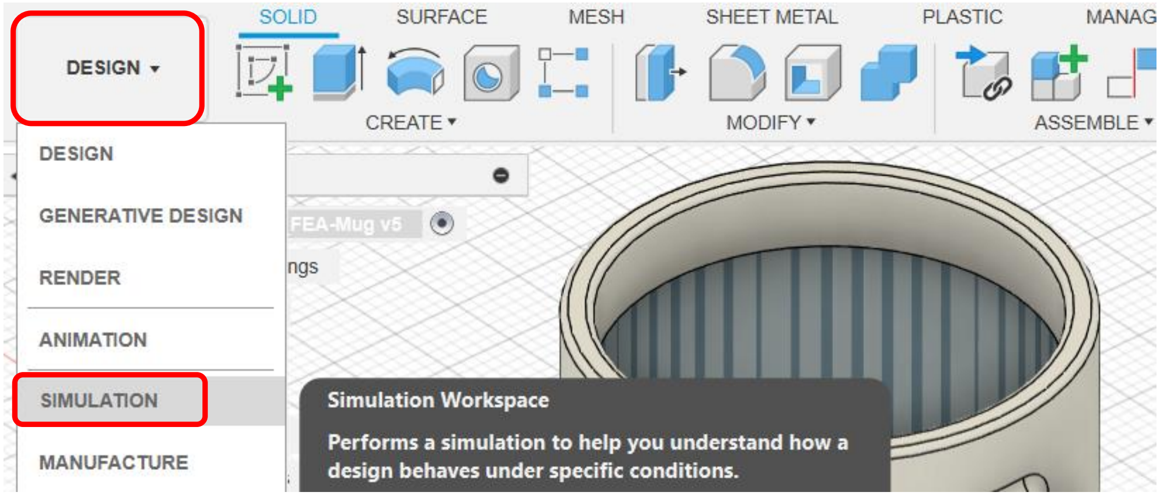


- click on the **eye** icon for the **Mug** Component to make the mug visible again

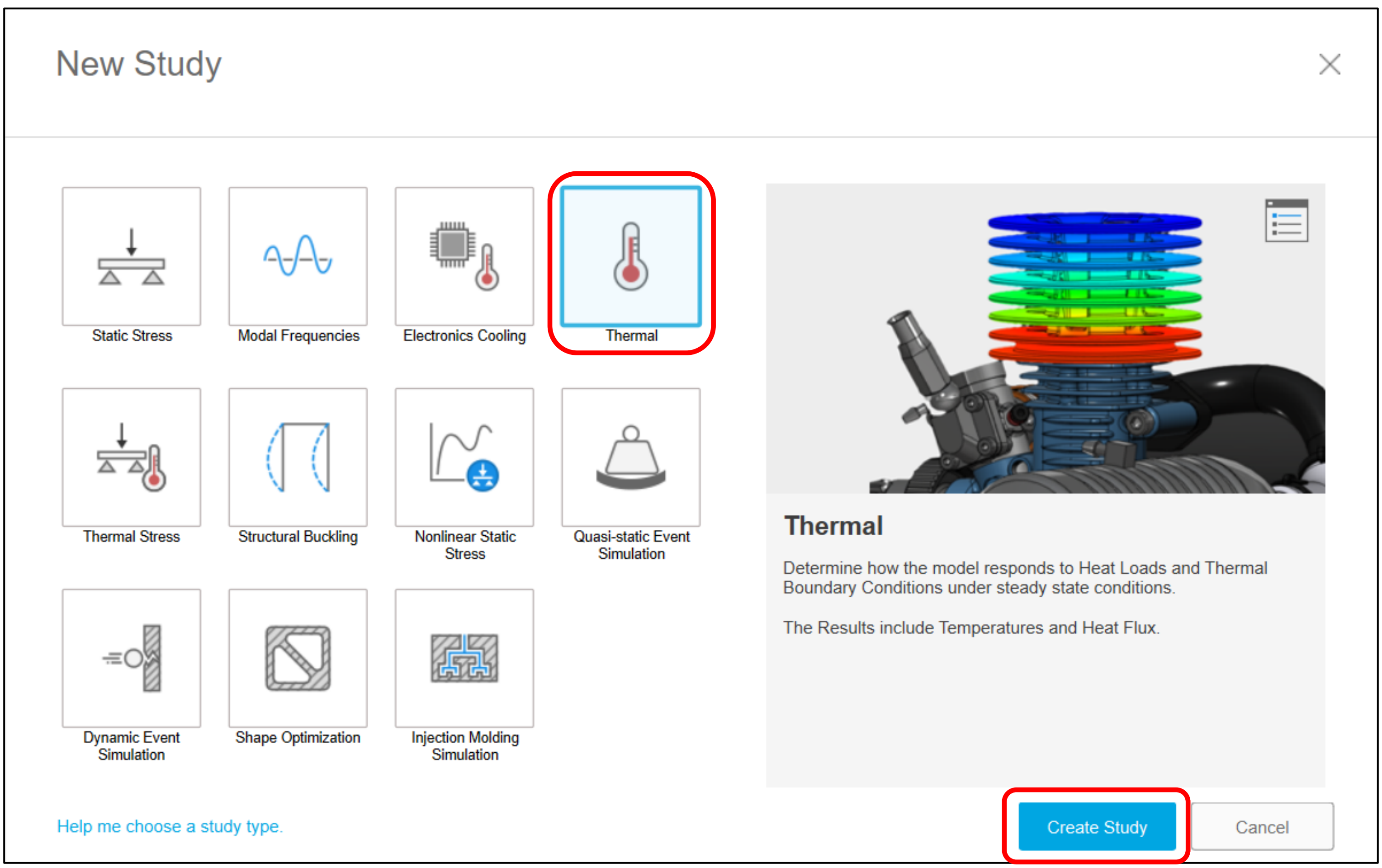


Performing a Thermal Simulation

- change the Workspace from **DESIGN** to **SIMULATION**

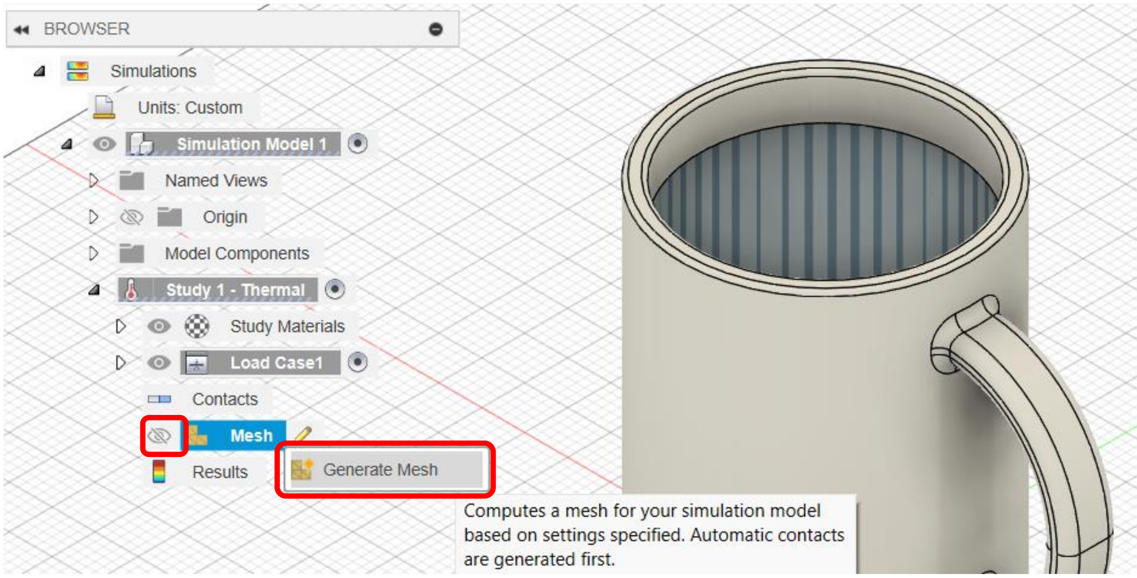


- marvel at all the simulations that are available
- select **Thermal** and click **Create Study**

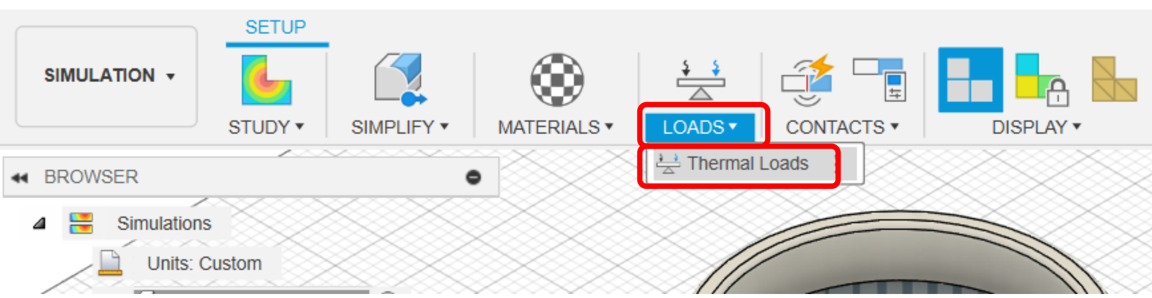


Setting a Thermal Load for Applied Temperature

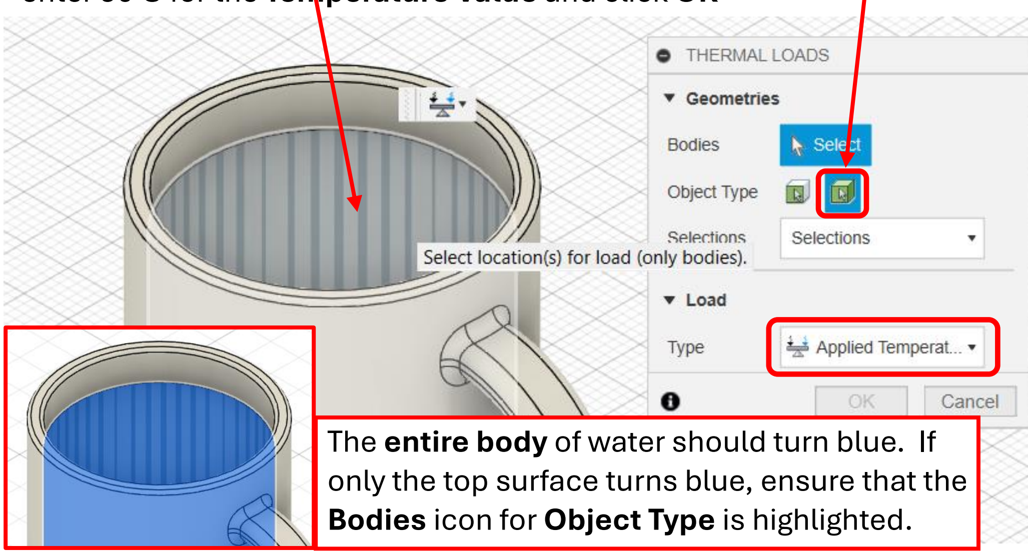
- right-click on **Mesh** and select **Generate Mesh** and wait a few seconds
- when the mesh shows, click on the **eye** icon to hide the Mesh

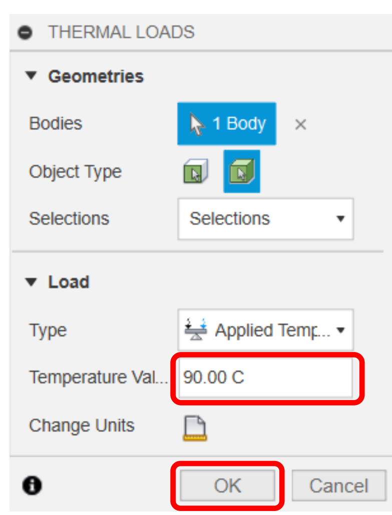


- from the **LOADS** menu select **Thermal Loads**



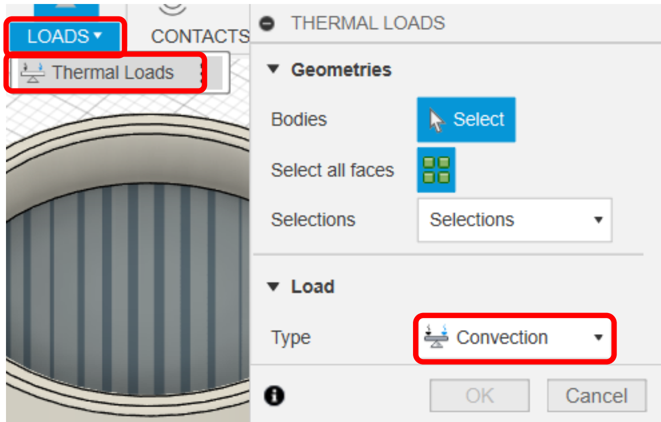
- change the **Type** to **Applied Temperature** and click on the **Bodies** icon for **Object Type**
- click on the **top surface** of the liquid, which should extend the window to show a **Temperature Value**
- enter **90 C** for the **Temperature Value** and click **OK**



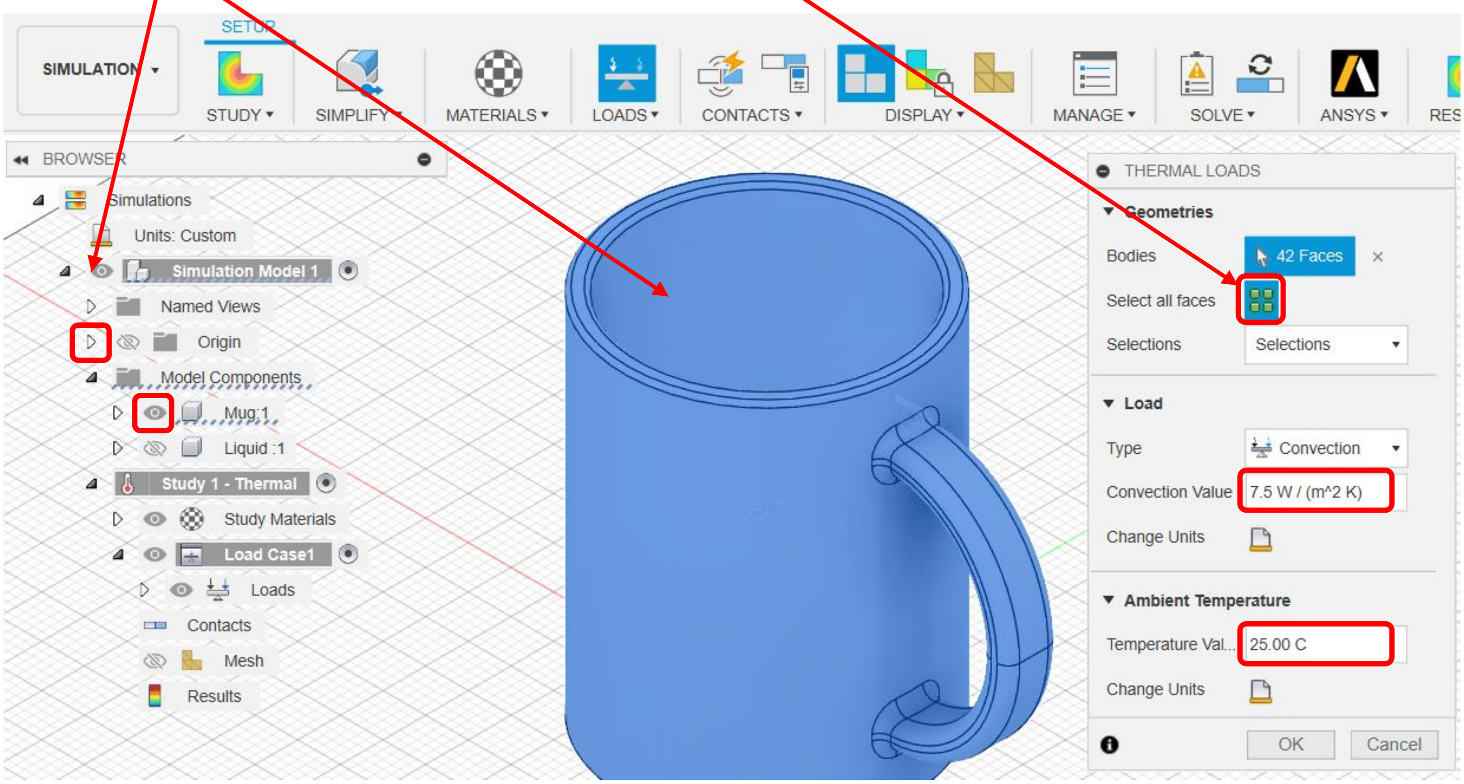


Setting a Thermal Load for Convection

- from the **LOADS** menu select **Thermal Loads** again and then for **Type** select **Convection**

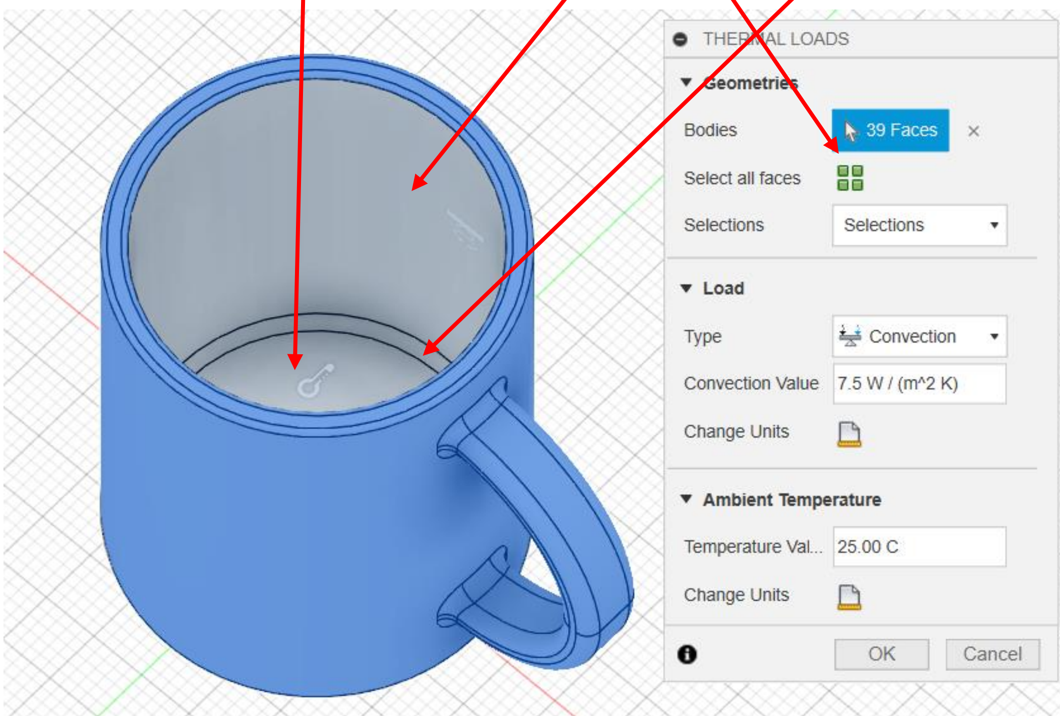


- click the **arrow** to open the **Model Components** folder and then click on the **eye** icon for the **Liquid** to hide it
- in the top Geometries section, click on the **Select all faces** icon to highlight it blue
- set the **Convection Value** to **7.5 W/(m^2 K)** and the **Ambient Temperature** to **25 C**
- click on the **Mug**, which should turn all of its faces blue. Do not click OK.

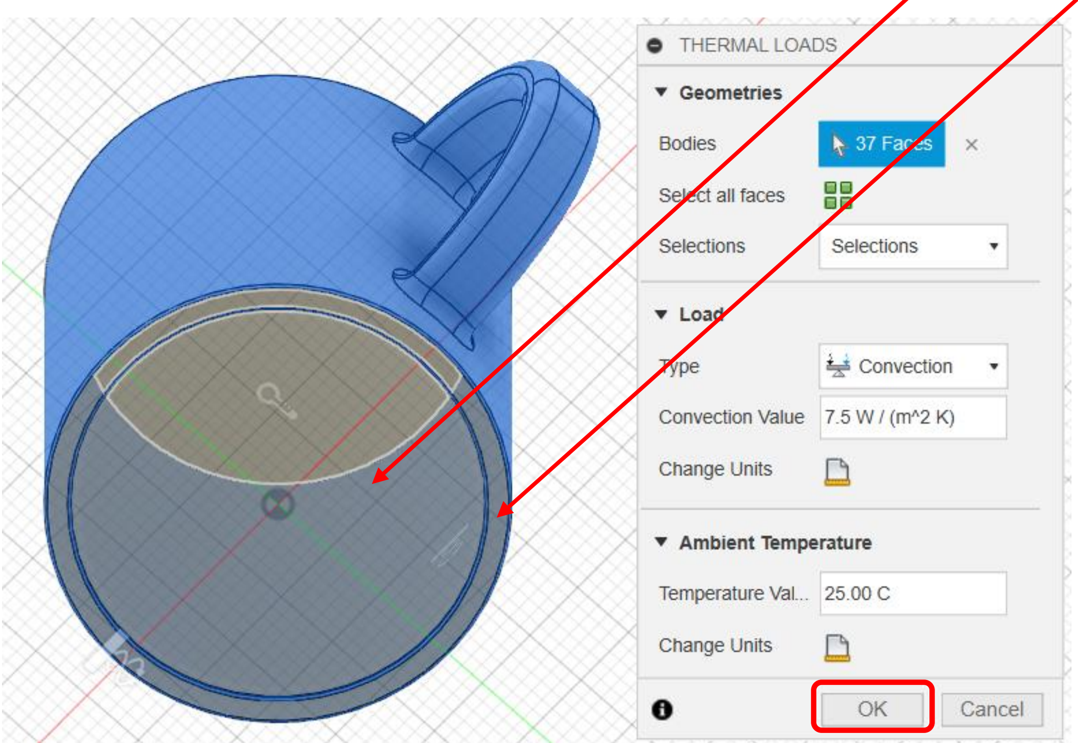


At this point all of the surfaces of the mug are set as “active” heat loss surfaces. Of course, the inner surface is covered by water, and the bottom surface doesn’t have any air flow and thus these surfaces do not contribute to convective cooling.

- adjust the view to look into the mug to access the bottom and inner walls
- in the top Geometries section, click on the **Select all faces** icon to remove its highlighting
- click on the **inner bottom surface**, the **inner wall**, and the **fillet** between them, which should remove the blue highlighting from these surfaces

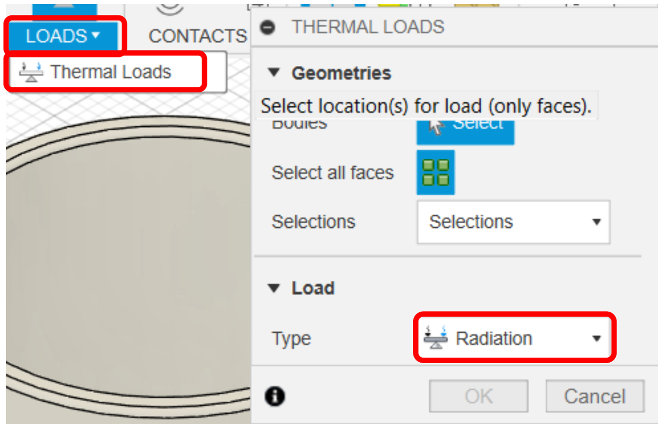


- adjust the view to access the bottom of the mug and click on the **bottom** and **outer ring** surfaces to remove their highlighting and click **OK**

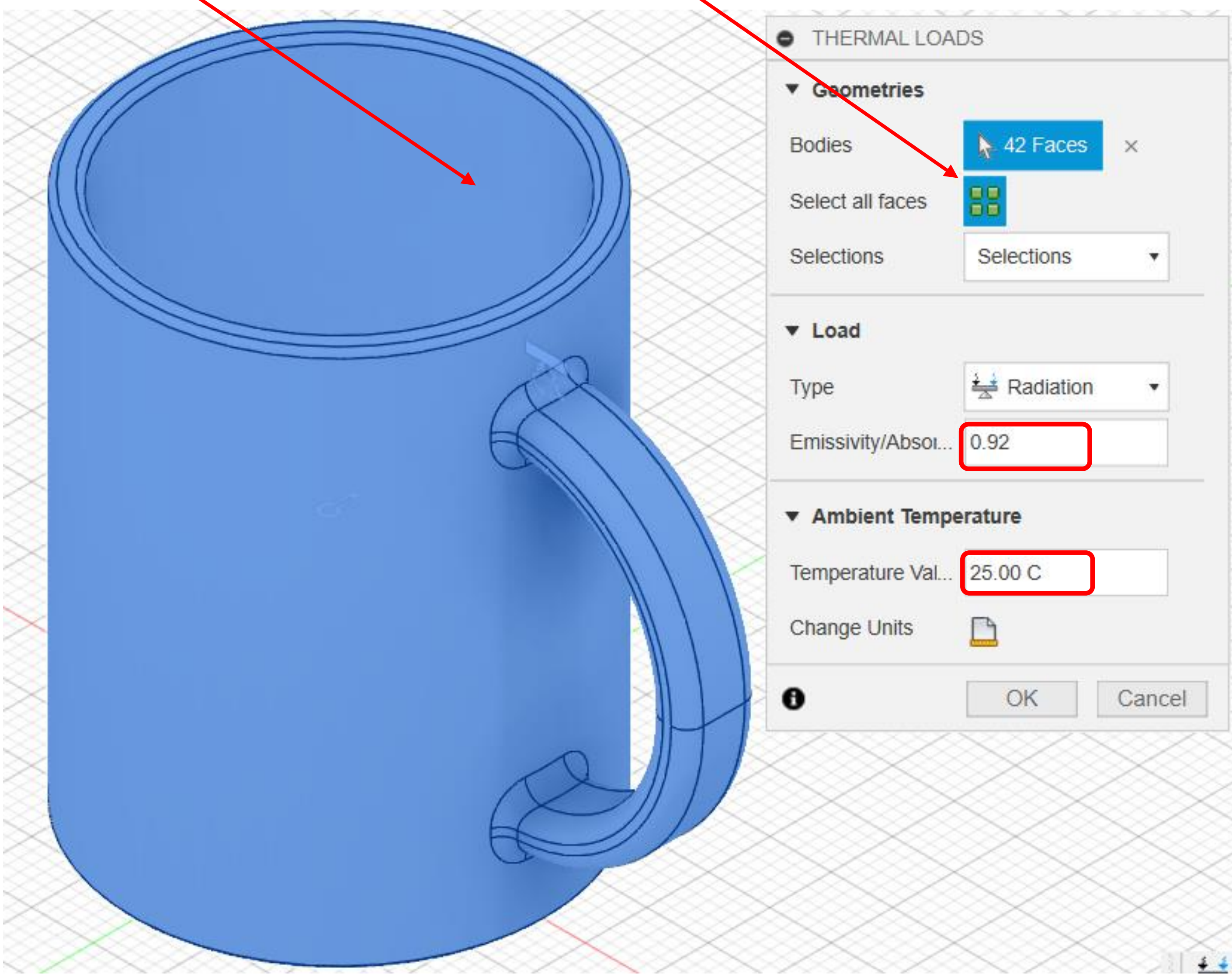


Setting a Load for Radiation

- from the **LOADS** menu select **Thermal Loads** again and then for **Type** select **Radiation**

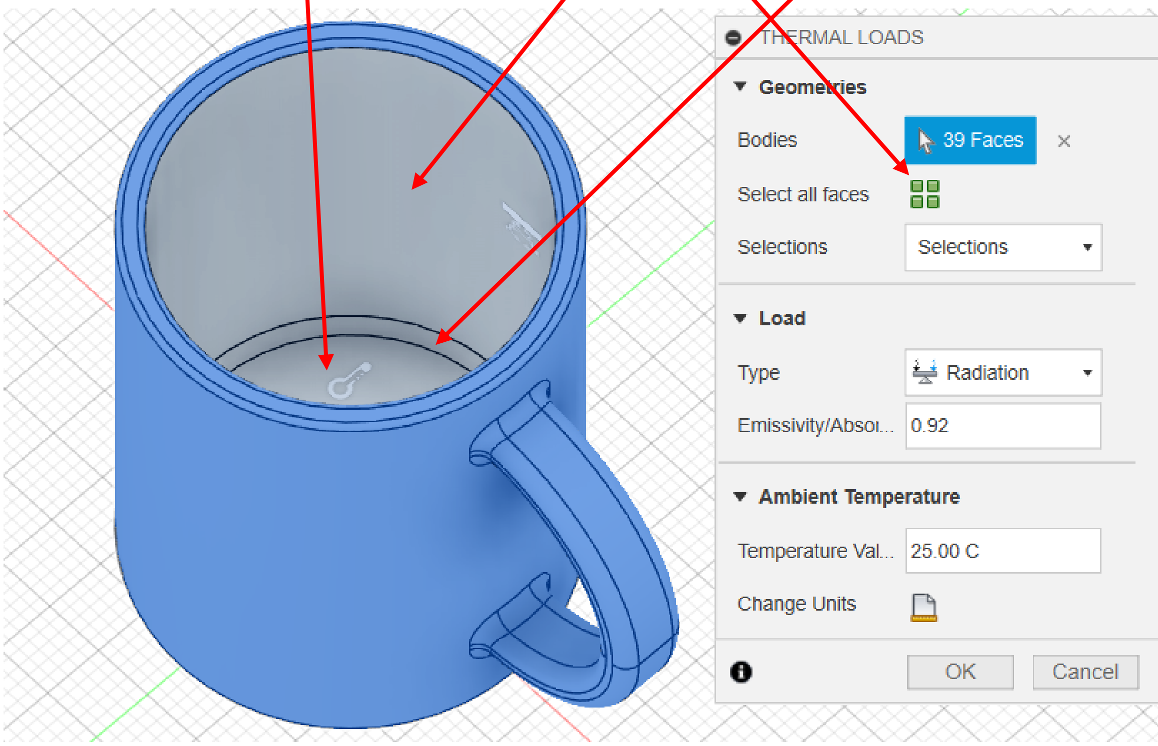


- in the top Geometries section, click on the **Select all faces** icon to highlight it blue
- set the **Emmissivity** to **0.92** and the **Ambient Temperature** to **25 C**
- click on the **Mug**, which should turn all of its faces blue. Do not click OK.

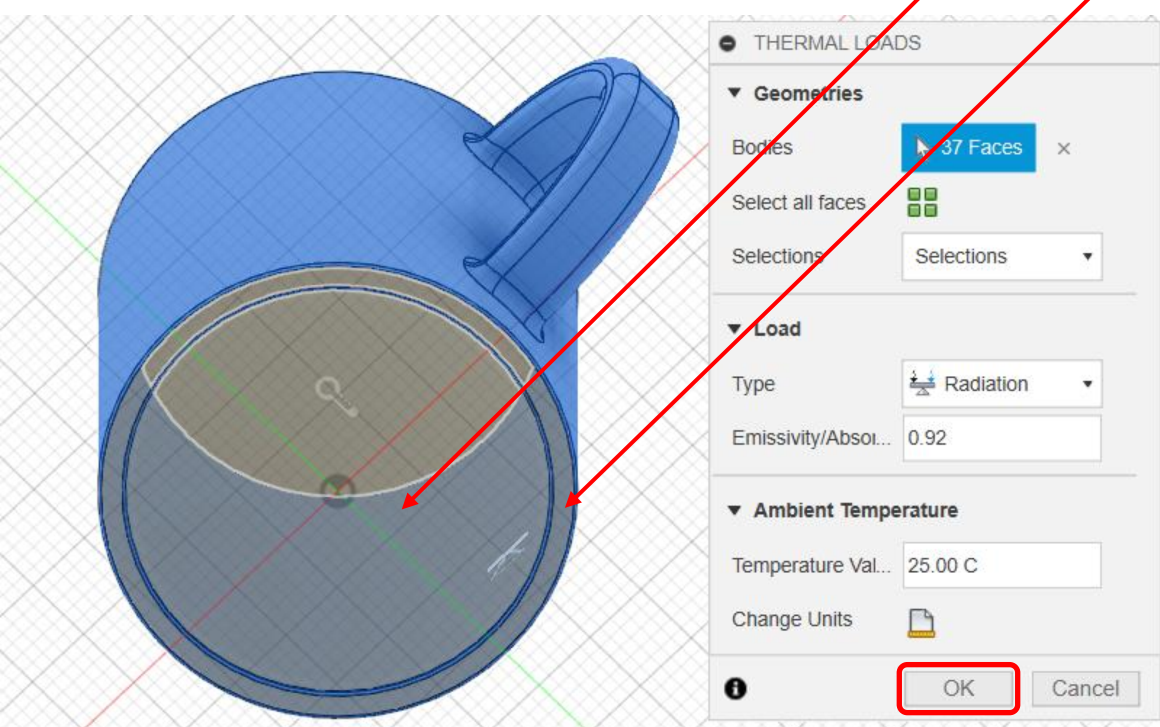


As we did for the Convection Load, we want to deactivate the inner and bottom surfaces that will not contribute to the radiation cooling.

- adjust the view to look into the mug to access the bottom and inner walls
- in the top Geometries section, click on the **Select all faces** icon to remove its highlighting
- click on the **inner bottom surface**, the **inner wall**, and the **fillet** between them, which should remove the blue highlighting from these surfaces

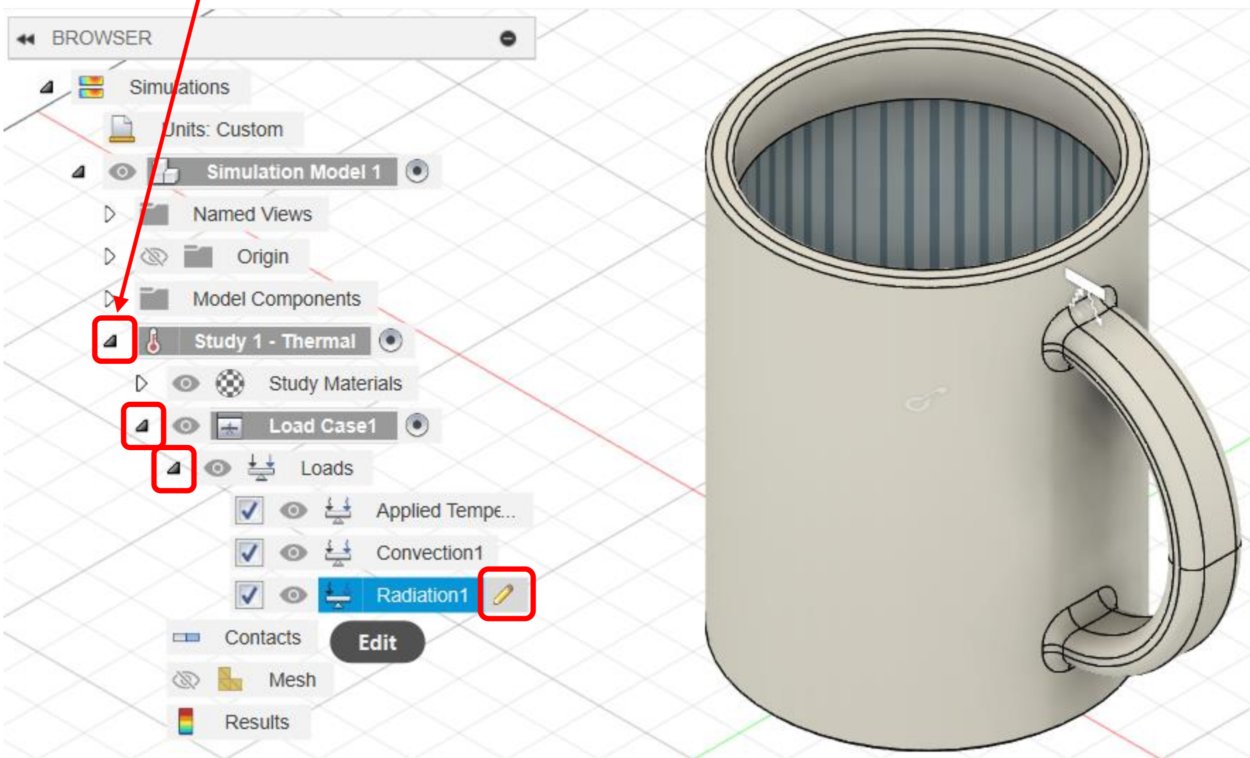


- adjust the view to access the bottom of the mug and click on the **bottom** and **outer ring** surfaces to remove their highlighting and click **OK**

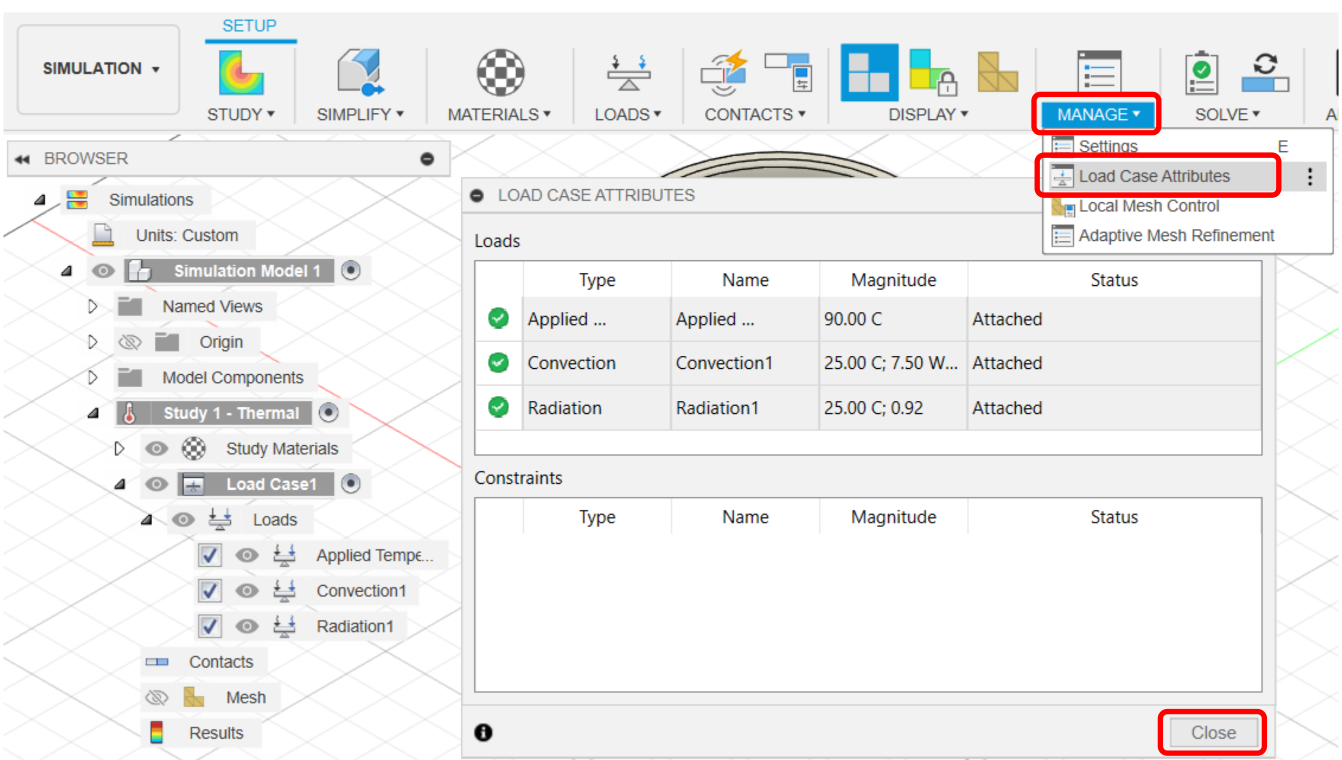


Editing and Viewing Thermal Loads

- click on the **arrows** to open the **Study**, **Load Case**, and **Loads**
- when hovering over the end of a load, one can click the **pencil icon** to revisit the settings for thst load
- one can also click on the Check boxes to disable a load. For now keep all checked. Later on, one may wish to turn off a load. For example, turning off radiation to determine its contribution to cooling.

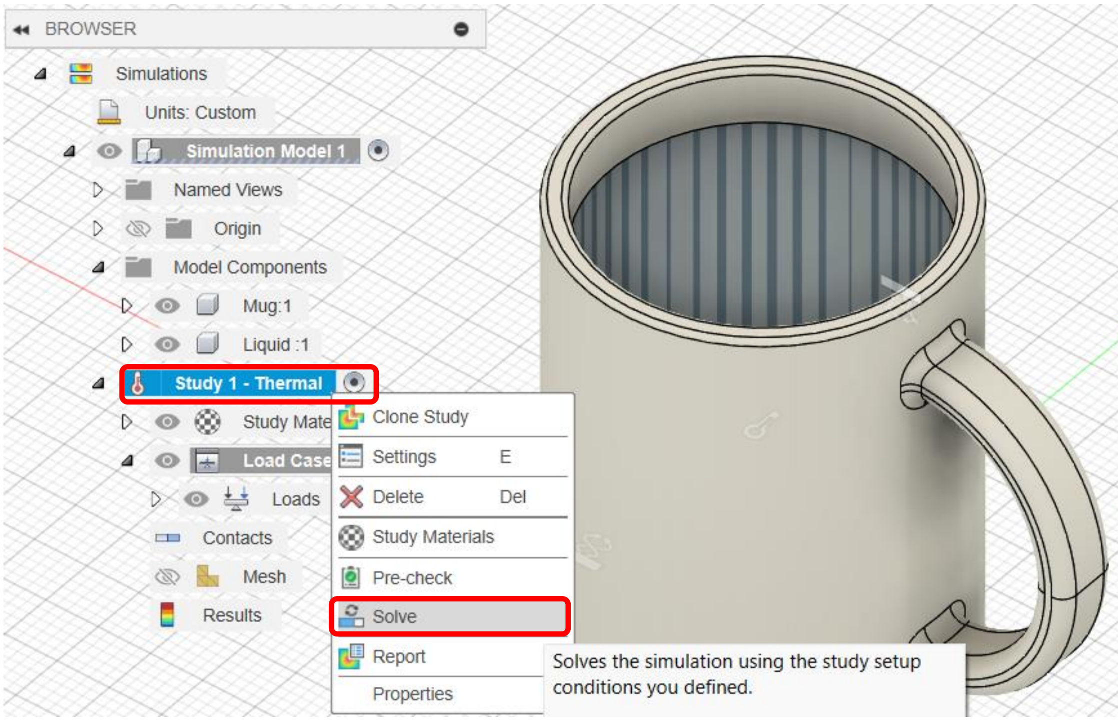


- from the **MANAGE** menu, select **Load Case Attributes** to open its window
- note the various loads and their values and click **Close**

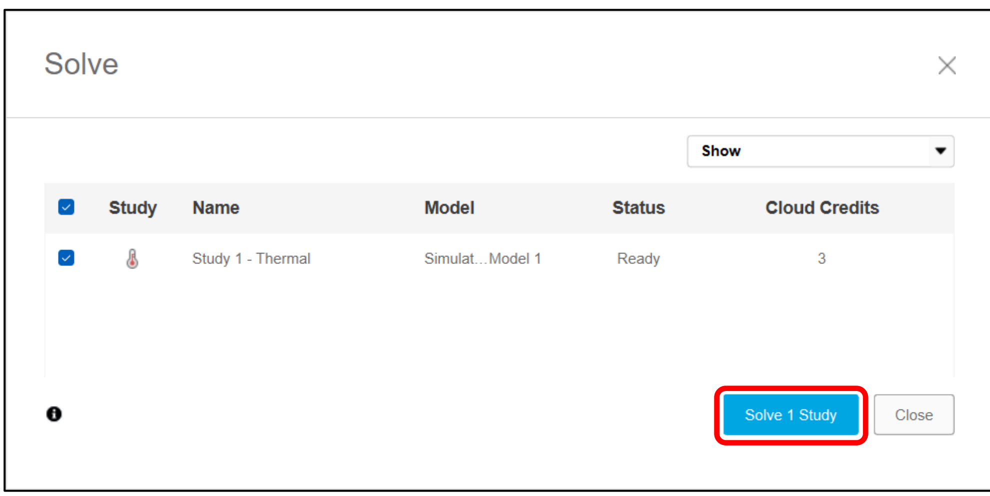


Running the Simulation

- right-click on the Study name and select **Solve**



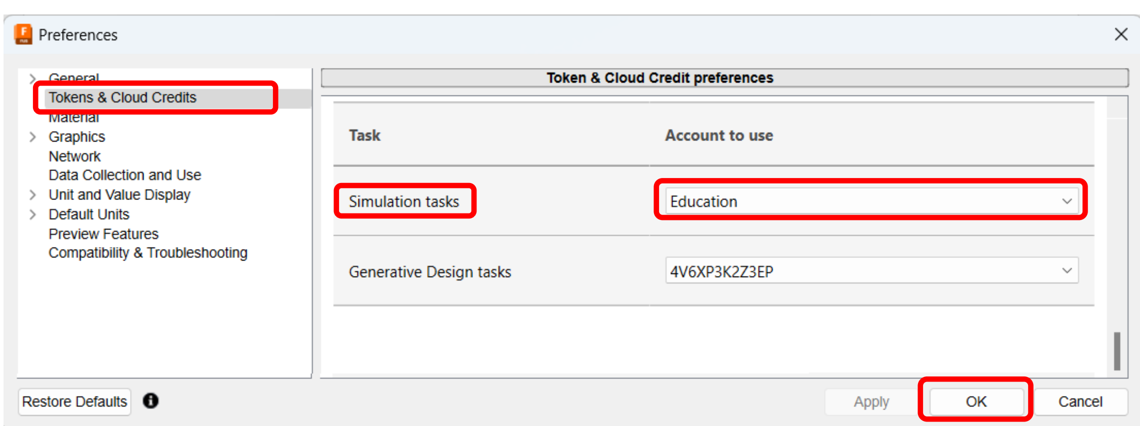
- click on the bottom **Solve 1 Study**. A simulation like this typically completes in about 2 minutes.



If a message about **Cloud Credits** opens, there are two options.

1) Transfer **V-Bucks** or **Robox** funds to Autodesk

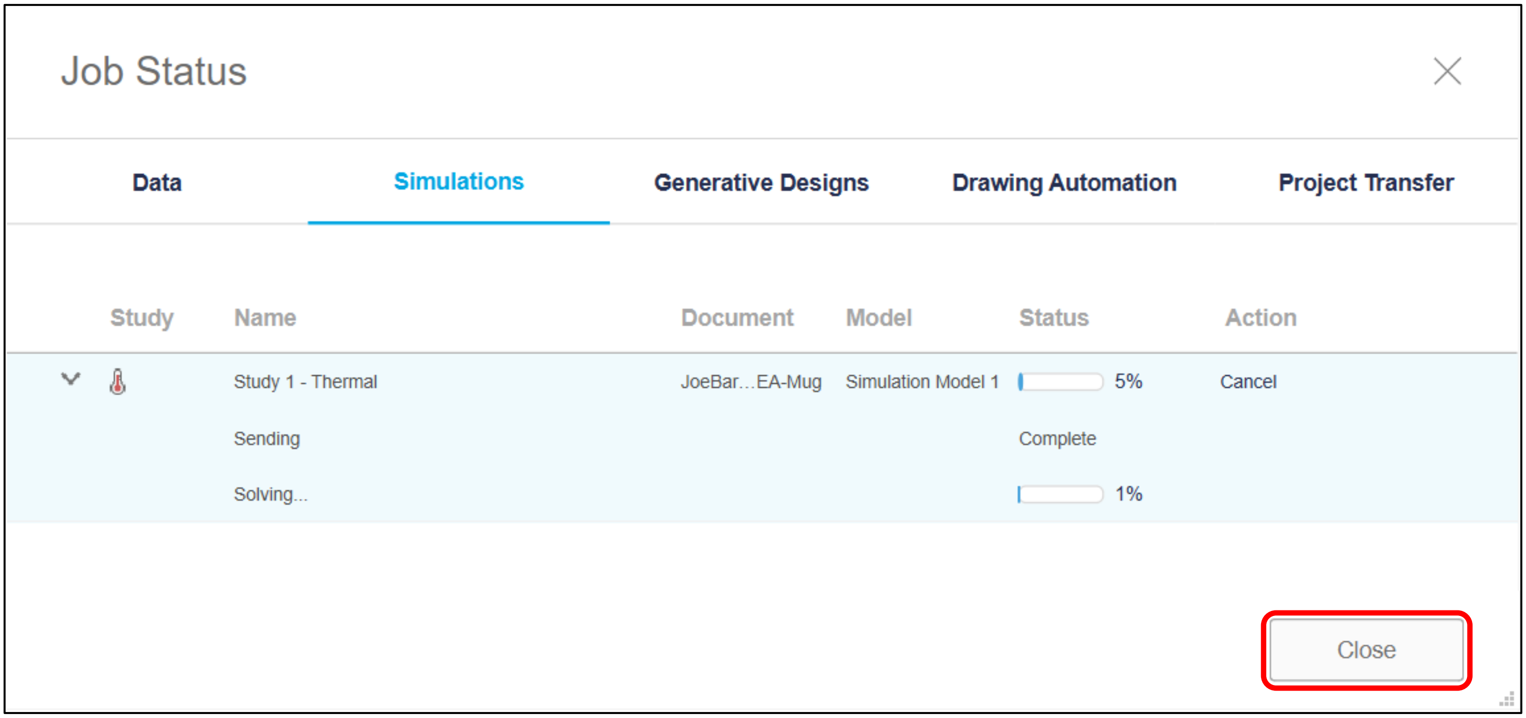
2) from the **top-right person icon of the Fusion screen**, select **Preferences**. Select **Tokens & Cloud Credits** on the left and for **Simulation tasks** select **Education**.



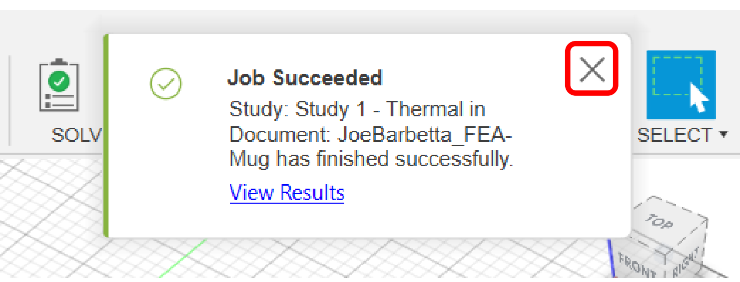
This screen will open to report the progress of the solve.

FEA simulations are computationally intensive and your project is sent to Autodesk servers in the “cloud”. The number crunching is performed on their servers.

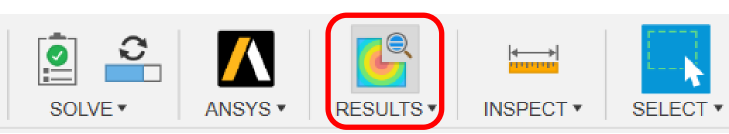
- when the simulation is finsihed, click **Close**



- when this message shows, click on the **X** to close it. Sometimes clicking on View Results does nothing.

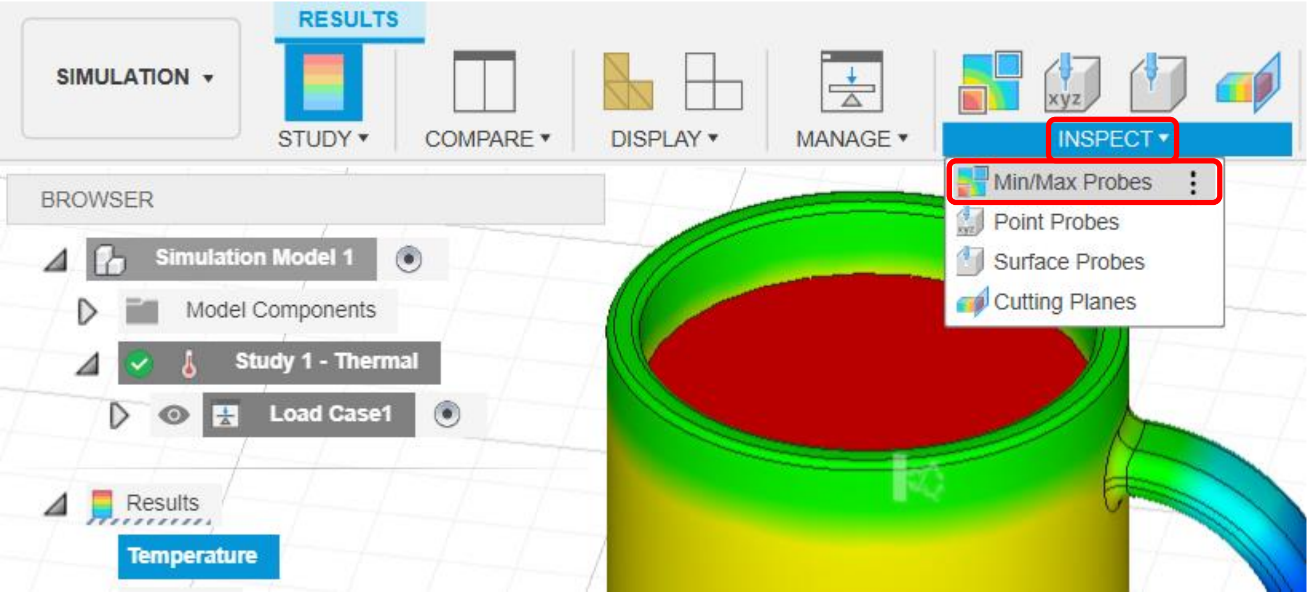


- click on the **RESULTS** icon and wait for the animation, shown on the right, to complete

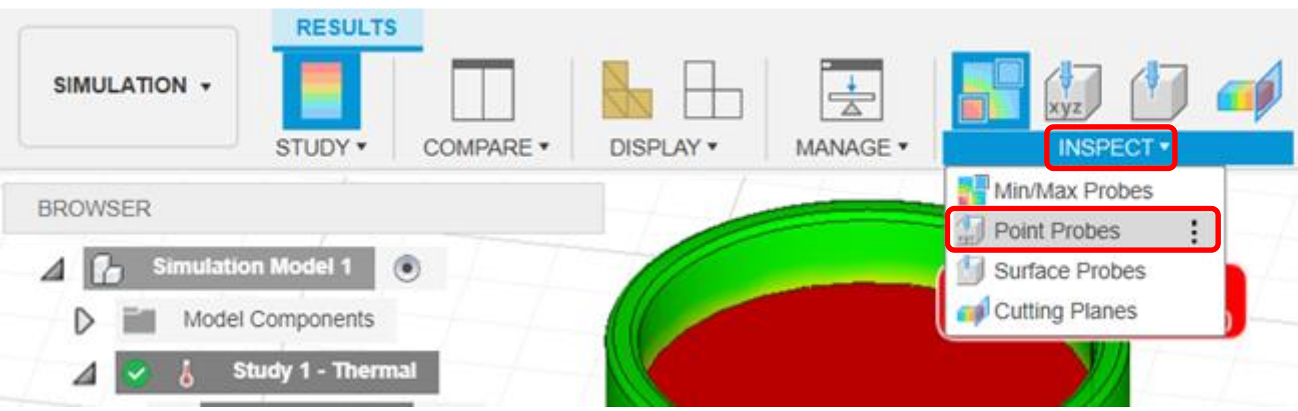


Adding Thermal Probes

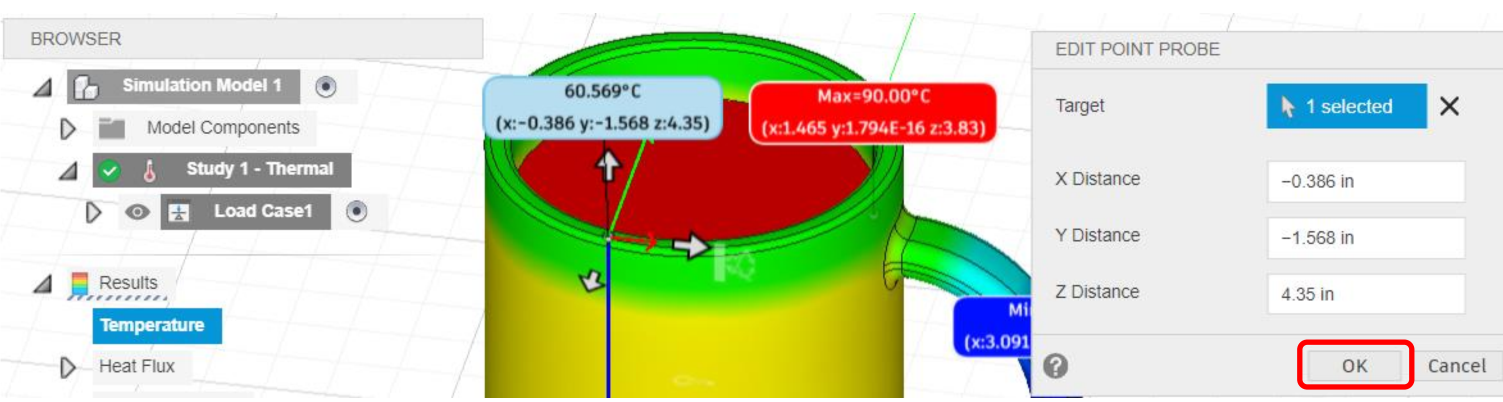
- from the **INSPECT** menu select **Min/Max Probes**



- from the **INSPECT** menu select **Point Probes**

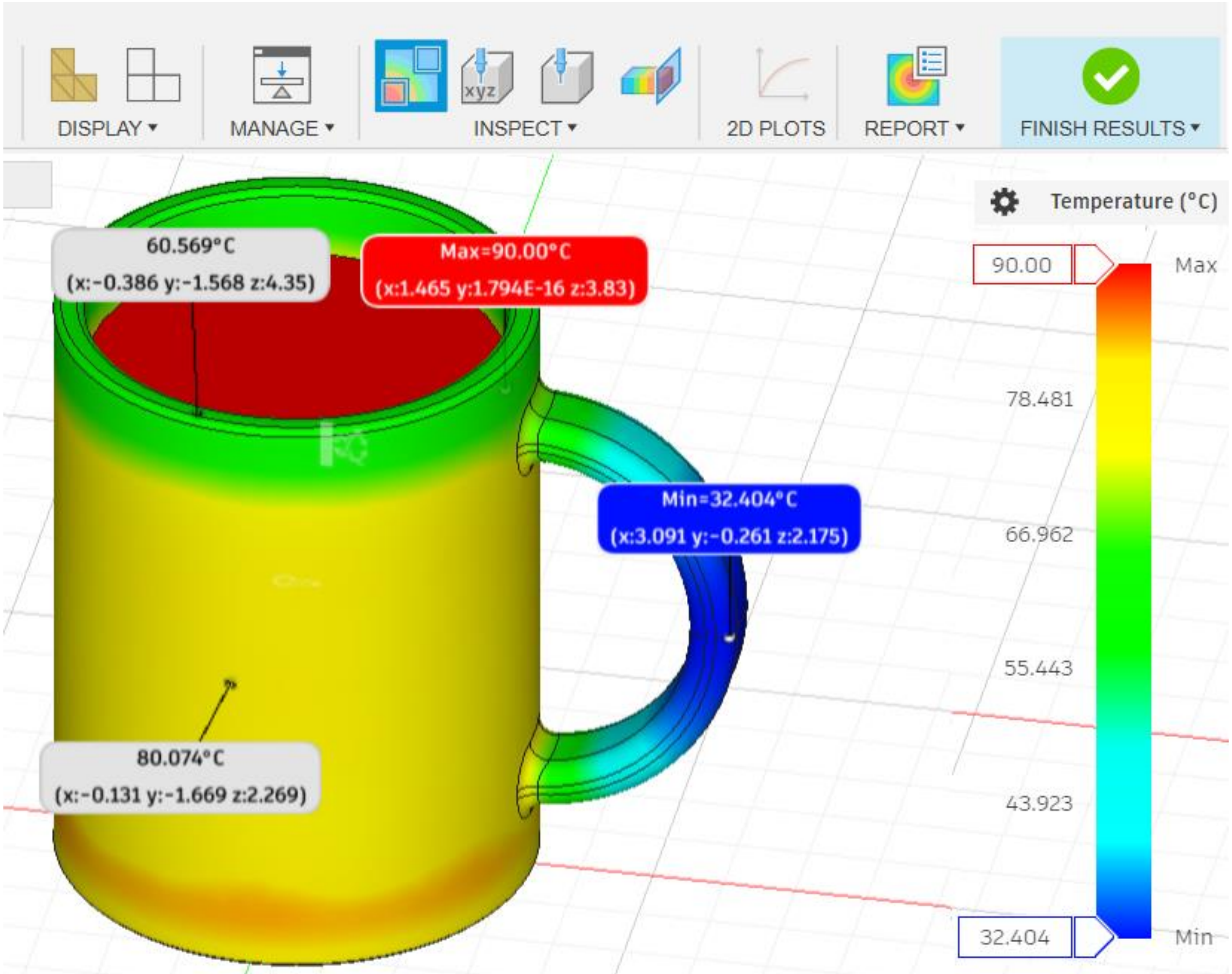


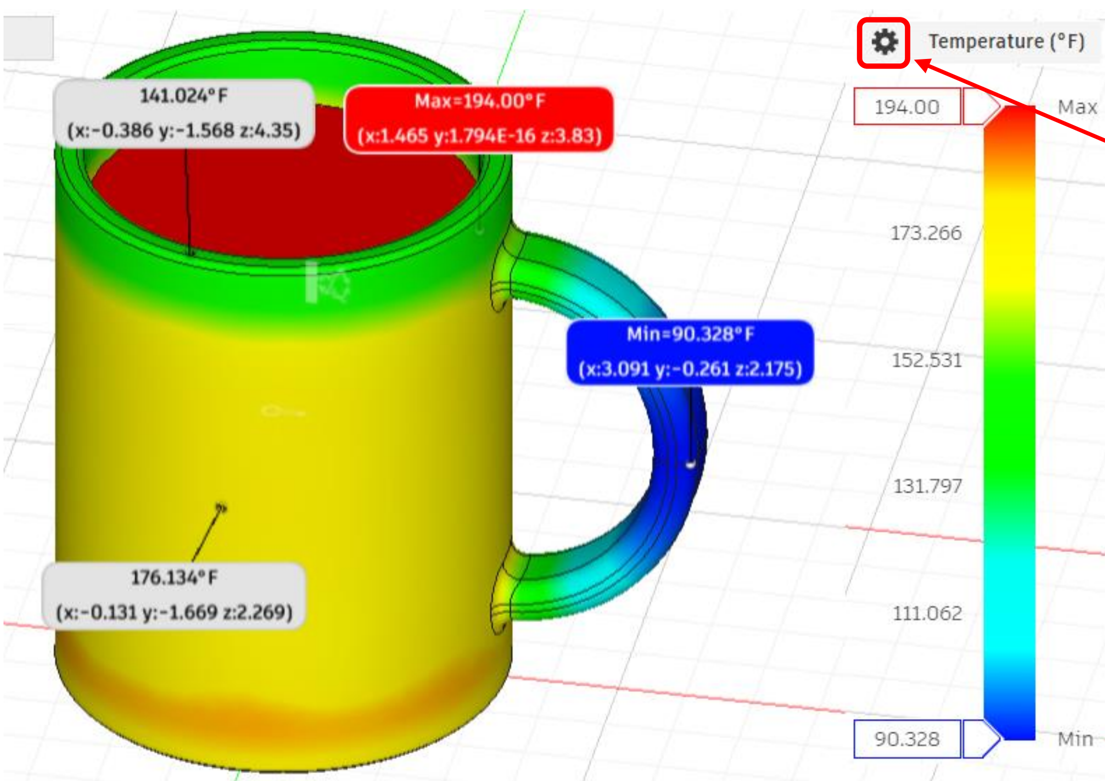
- click anywhere on the **top rim** of the mug and click **OK** on the **EDIT POINT PROBE** window. The temperature and **Distance** values will be different and the **Distance** values do no need to be changed.



- create another **Point Probe** somewhere on the **side of the mug**

Below is the result of the simulation and the probes. This screenshot has been edited to bring the scale closer to the mug.

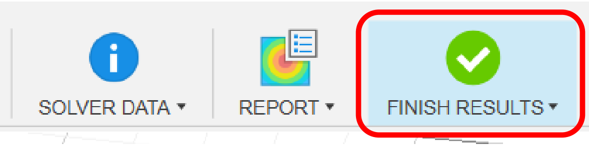




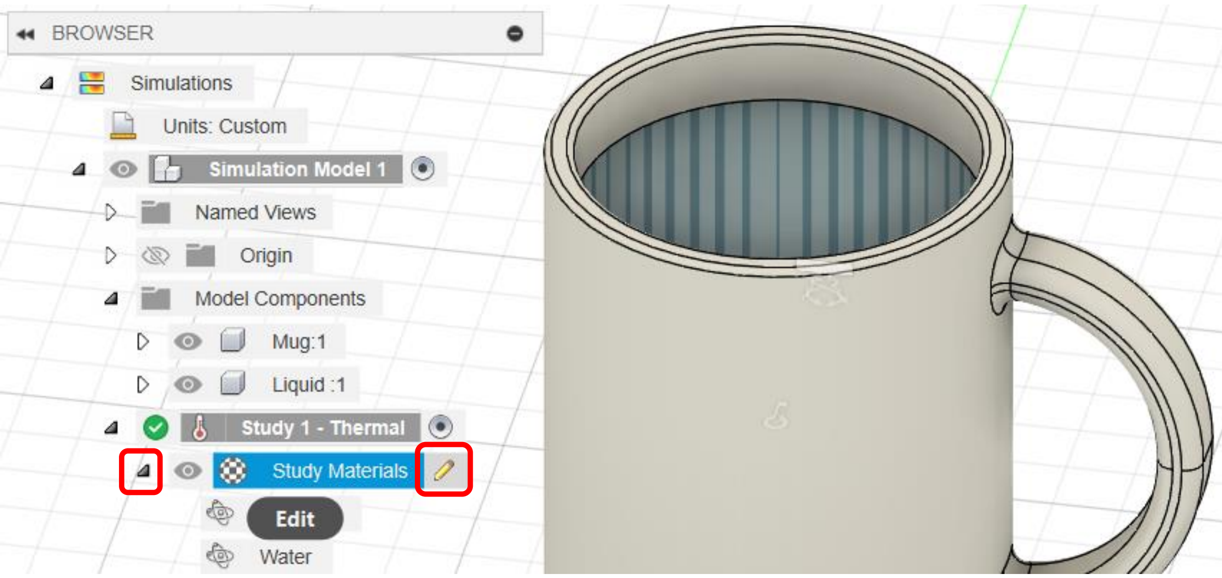
This is result after clicking on the gear icon and selecting **F**.

Changing Study Materials

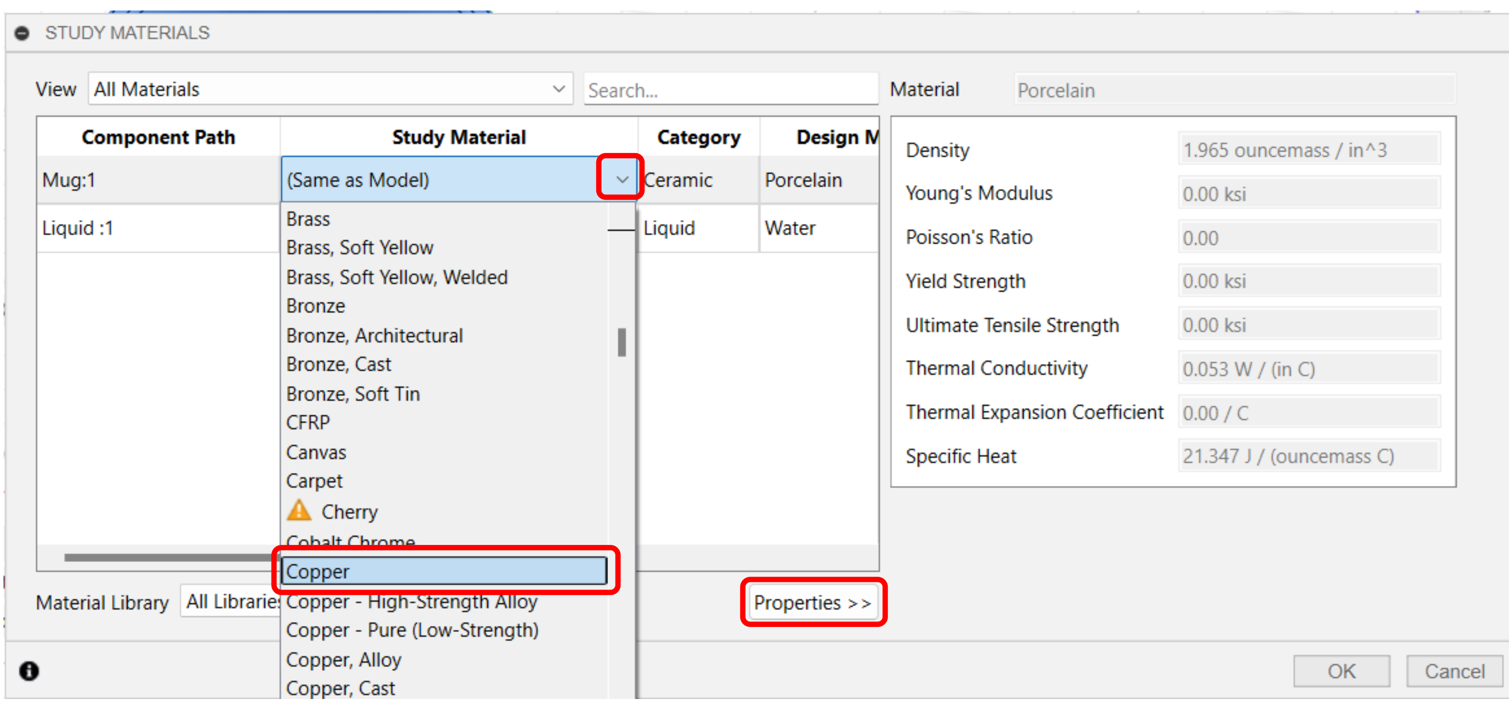
- click on **Finish Results** at the top right of the Fusion screen



- click on the arrow for **Study Materials** and then hover over **Study Materials** and click the **Pencil icon**



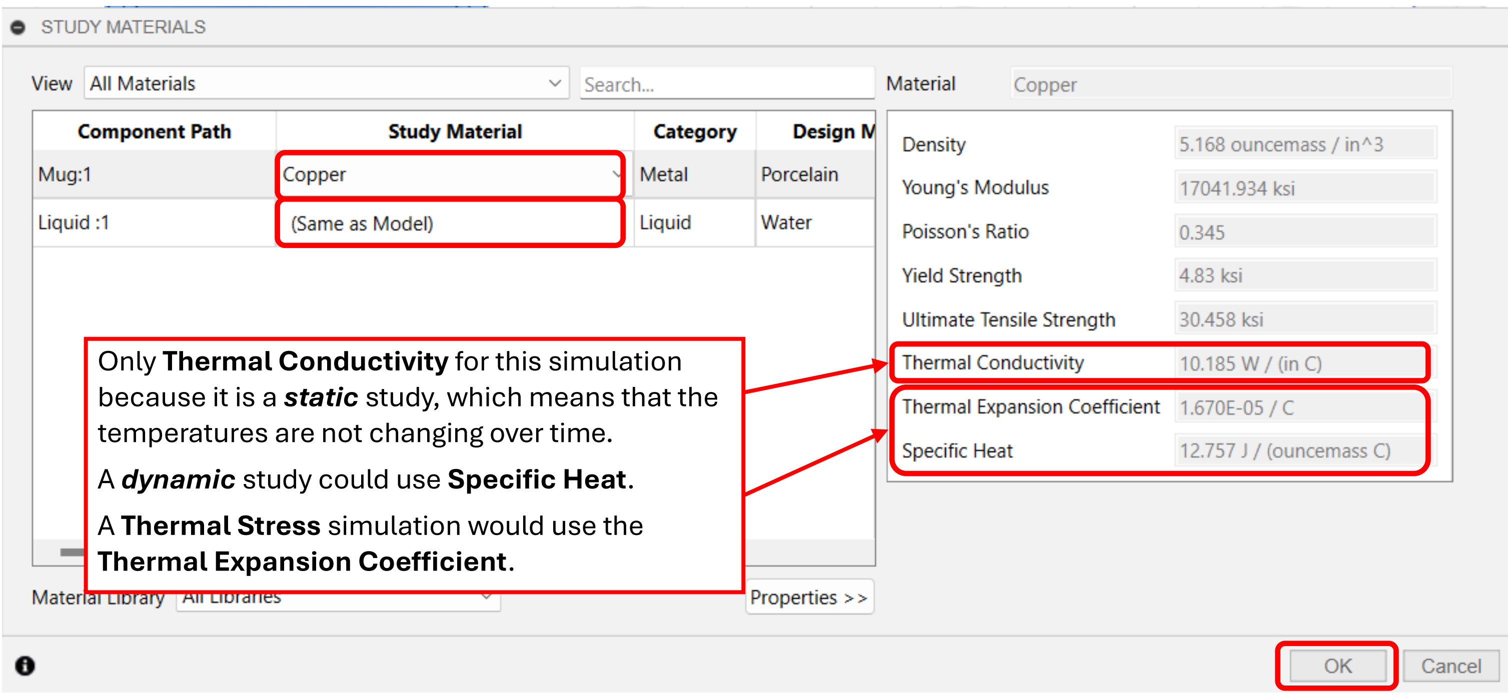
- click on the **Properties > >** button to open the Material properties of the material
- click on the Mug material and click on the **arrow icon** for the pull-down menu
- scroll down and select **Copper**



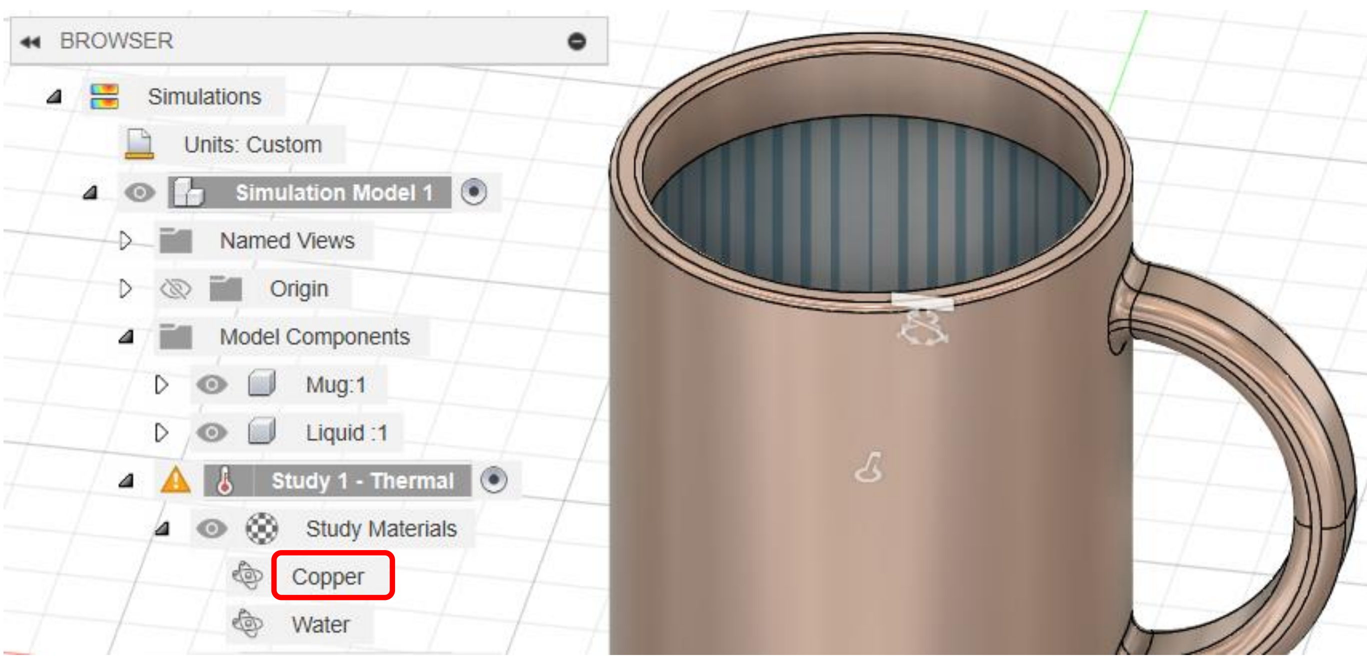
- click on (Same as Model) and then click on Copper to force the Material properties to update.

The **Thermal Conductivity** shows as **10.185 W/(in*C)** vs **0.053 W/(in*C)** for Porcelain. Fusion is using units that are a mix of metric and inches. By multiplying by 39.37 in/m, inches are cancelled out (make the light saber sound) and the results are **401 W/(m*C)** for copper and **2.09 W/(m*C)** for Porcelain. Note that is common to see units as **W/(m*K)**. Because thermal conductivity is based on a **temperature difference C (Celcius) and K (Kelvin) are interchangable**. This is not true for other uses of temperature. For example, when using the Ideal Gas Equation P*V = n * R * T, one must ensure the units for T are Absolute Temperature and thus K must be used.

- click **OK**

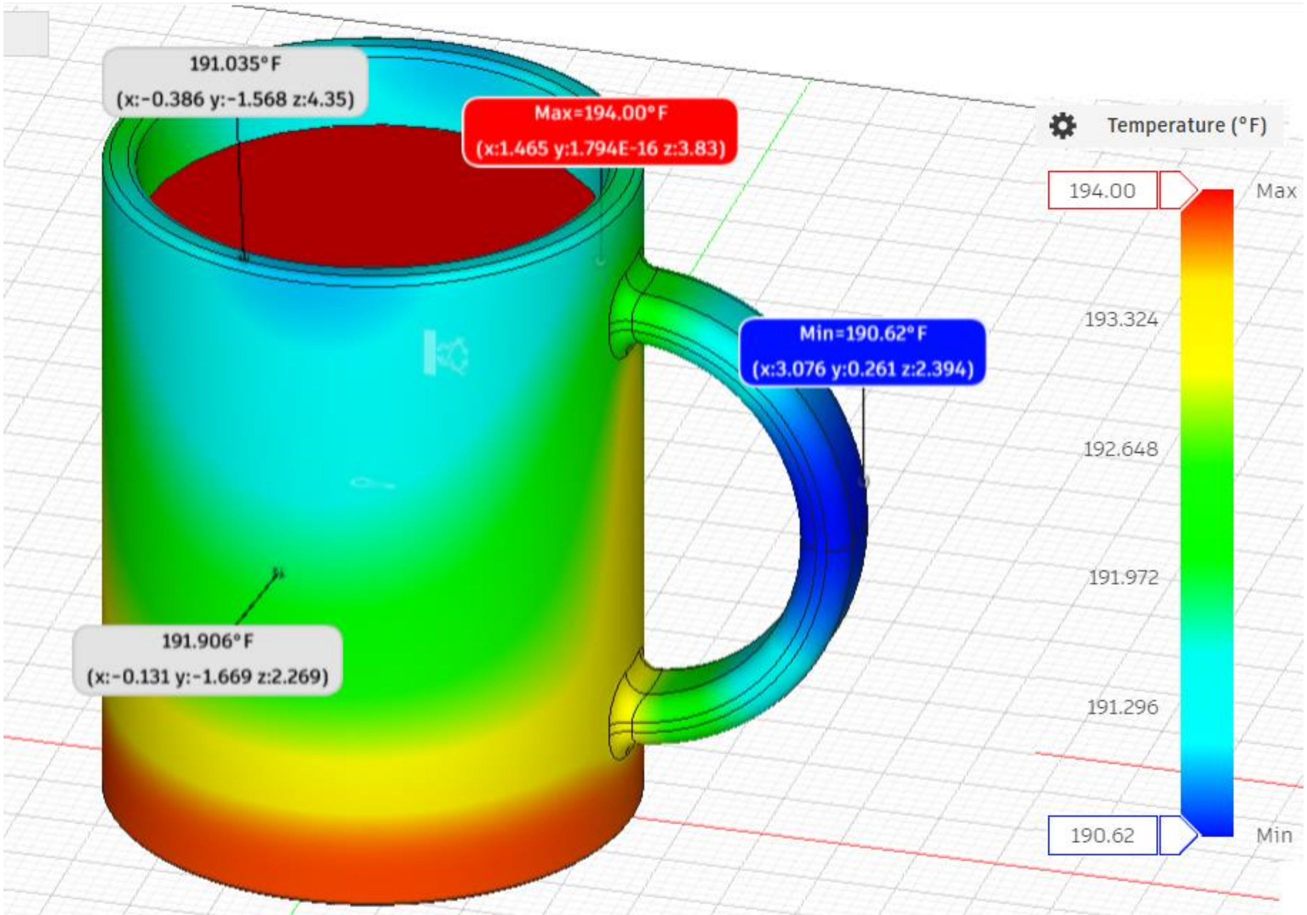


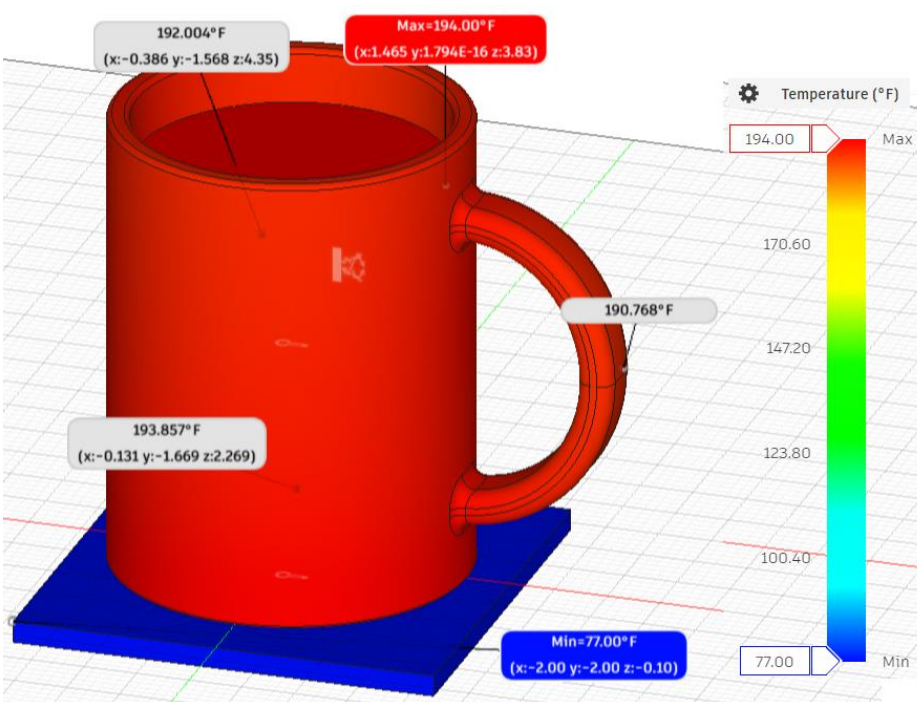
Note that Copper now shows as a study material.



The minimum temperature, which is at the middle of the handle, is now **100 F higher** with a copper mug. The handle temperature was **90.3 F** with a **porcelain** mug. Here is is **190.6 F** with a **copper** mug.

Note that the **colors my be deceiving.** Dark blue represents the coolest temperature of the study and Not ambient temperature. The blue 190 F temperature is only 4 F cooler than the 194 F water temperature. It is still hot.





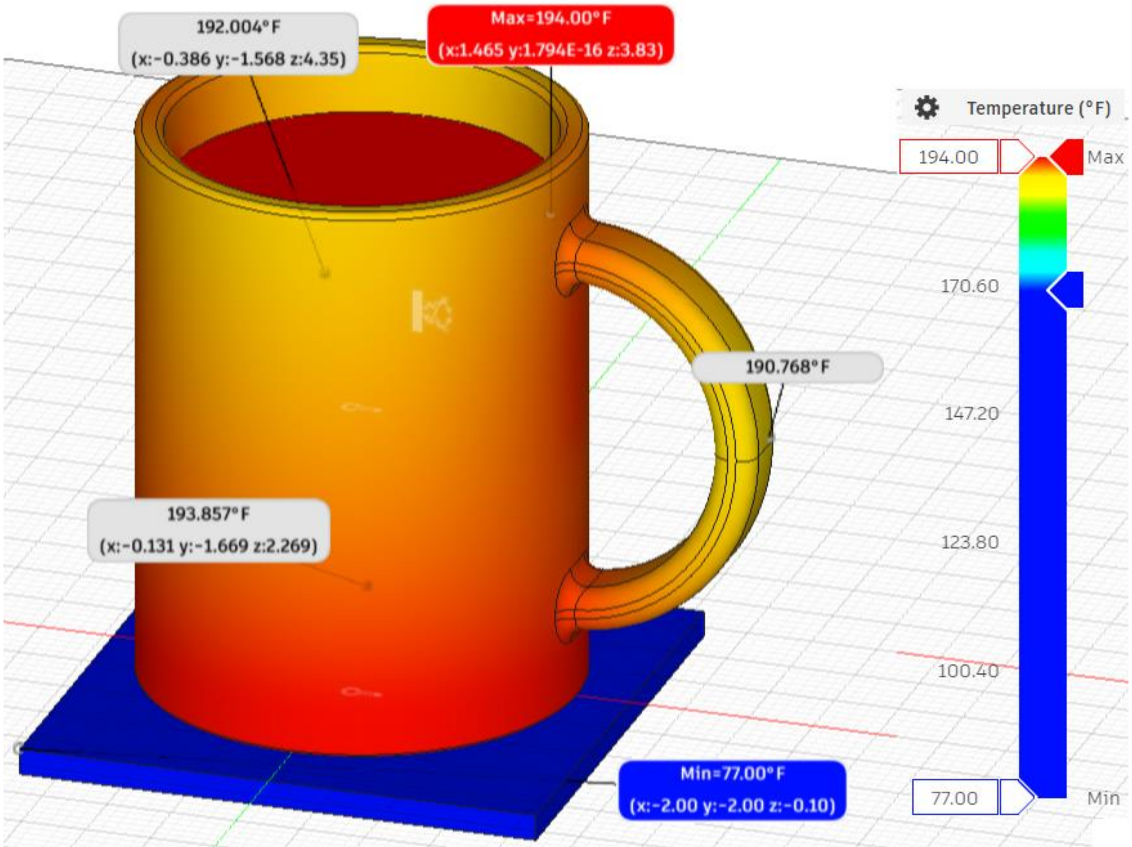
This is an example where a plate was added to the simulation to "fix" the desireed low temperature. It is not in thermal contact with the mug and had an **Applied Temperature** of **25 C (77 F)** set.

The problem with this coloring is that one cannot discern the lower temperature of the handle by its color.

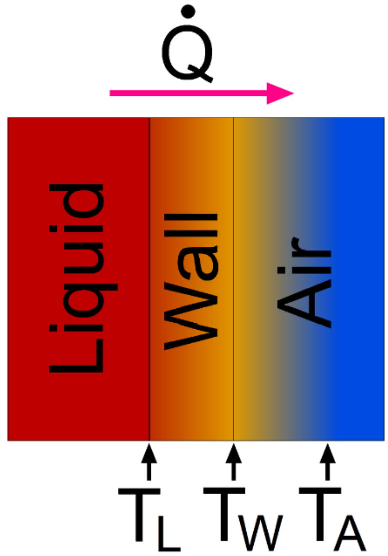
This does not need to be done for the assignment.

One can make the “cold plate” method better by clicking on the **gear icon** for Temperature and checking the **Show Threshold Slider** option. The slider that appears at the bottom right of the temperature scale can be dragged upward to achieve the desired coloration. In this case it was dragged to a point to see some color variation.

This does not need to be done for the assignment.



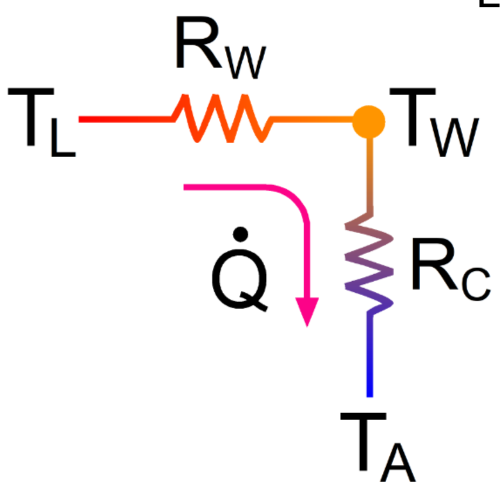
Thermal Circuit Analysis



This illustrates heat flowing throught the wall of the mug.

Note that there is a **temperature gradient** through the wall and in the air at the wall surface.

Equivalent circuit to calculate $\mathbf{T_W}$ and $\mathbf{\dot{Q}}$



$\mathbf{T_L}$ = Temperature of Liquid
$\mathbf{T_W}$ = Temperature of Wall (Outer)
$\mathbf{T_A}$ = Ambient Temperature
$\mathbf{R_W}$ = Thermal Resistance of Wall
$\mathbf{R_C}$ = Thermal Resistance of Convection
$\mathbf{\dot{Q}}$ = Heat Flow

Ohmelli's law

$$V = I\ R$$

Thermelli's law

$$\Delta T = \dot{Q}\ R$$

Note that the Thermal Conductivity value shown by Fusion is a mix of metric and English units. Here is the value after converting inches to meters to see the full metric value to compare to values published on-line. We will continue to use the inches value in our calculations because all inch units will cancel out.

But, isn't that dangerous? Didn't a metric English unit misup cause the $100 million Mars Climate Orbiter to burn up in the Martian atmosphere in 1999? Dude, This is just class assignment.

$$k = \mathbf{0.053}\ \frac{W}{in\ C} \cdot \frac{1\ in}{0.0254\ m} = \mathbf{2.09}\ \frac{W}{m\ C}$$

Calculations for **R**$_W$ (Thermal Resistance of Mug Wall):

$$MugID = MugDiameter - 2 \cdot MugWallThickness$$

$$MugID = \mathbf{3.25}\ in - 2 \cdot \mathbf{0.21}\ in = \mathbf{2.83}\ in$$

$$A_{inner} = \pi \cdot MugID \cdot LiquidLevel$$

$$A_{inner} = \pi \cdot \mathbf{2.83}\ in \cdot \mathbf{3.5}\ in = \mathbf{31.1}\ in^2$$

$$R_W = \frac{L}{k\ A} = \frac{MugWallThickness}{k\ A_{inner}}$$

$$R_W = \frac{\mathbf{0.21}\ in}{\mathbf{0.053}\ \frac{W}{in\ C}\ \mathbf{31.1}\ in^2} = \mathbf{0.127}\frac{C}{W}$$

Calculations for **R**$_C$ (Thermal Resistance of Convection):

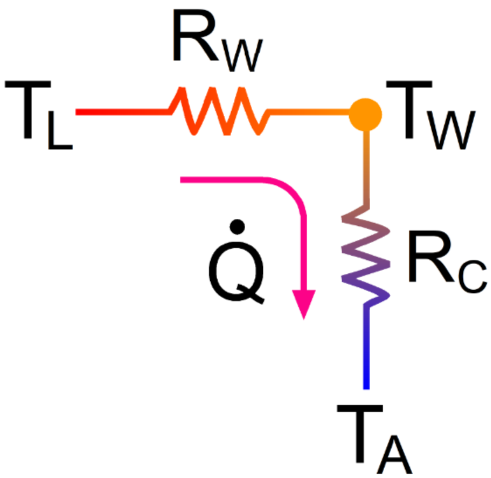
$$A_{outer} = \pi \cdot MugDiameter \cdot MugHeight$$

$$A_{outer} = \pi \cdot \mathbf{3.35}\ in \cdot \mathbf{4.35}\ in = \mathbf{45.8}\ in^2$$

$$R_C = (k_{Conv} \cdot A)^{\frac{1}{2}}$$

$$R_C = \left(\mathbf{7.5}\frac{W}{m^2C} \cdot \frac{1\ m^2}{1550\ in^2} \cdot \mathbf{45.8}\ in^2\right)^{\frac{1}{2}} = \mathbf{4.51}\frac{C}{W}$$

Calculations for T_W (Mug Wall Temperature):



$$\dot{Q} = \frac{T_L - T_A}{R_W + R_C}$$

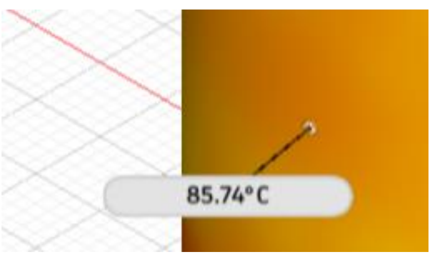
using Thermelli's law to determine heat flow

$$\dot{Q} = \frac{\mathbf{90}^\circ\,\mathrm{C} - \mathbf{25}^\circ\,\mathrm{C}}{\mathbf{0.222}\,\frac{C}{W} + \mathbf{4.51}\,\frac{C}{W}} = \mathbf{13.7}\,W$$

$$T_W = \dot{Q} \cdot R_C + T_A$$

using Thermelli's law to determine Wall Temperature

$$T_W = \mathbf{13.7}\,W \cdot \mathbf{4.51}\,\frac{C}{W} + \mathbf{25}^\circ\,\mathrm{C} = \mathbf{86.8}^\circ\,\mathrm{C}$$

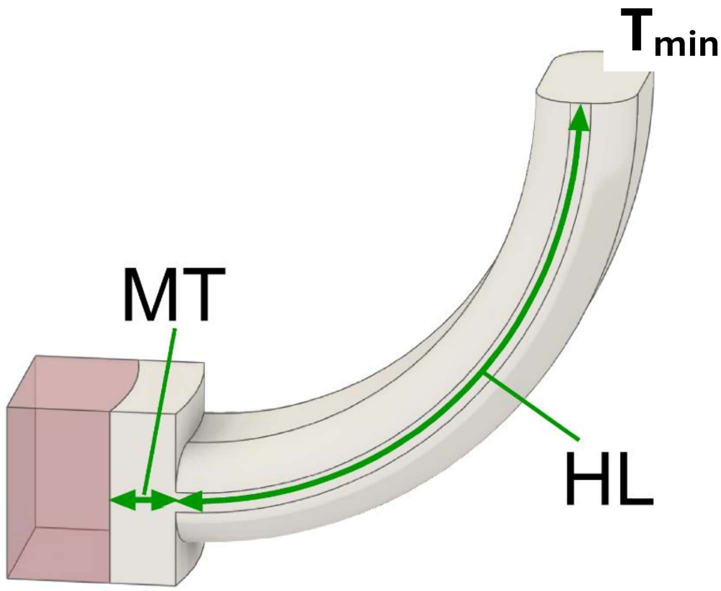


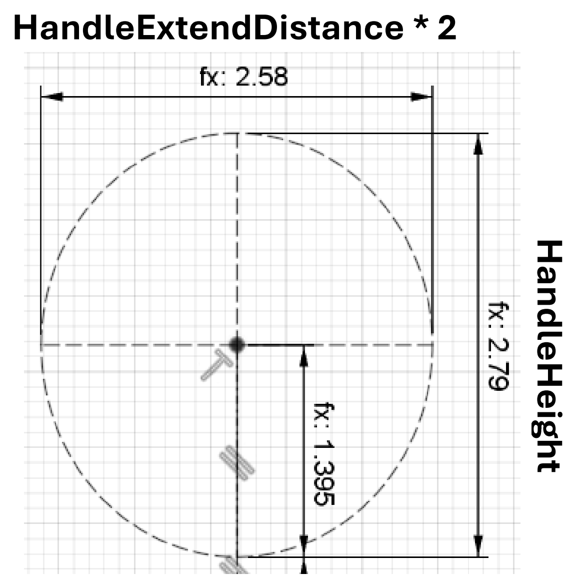
When we performed the simulation with **only convectional cooling** (radiational cooling turned off) the mug wall temperature was pretty close to our calcuation.

Isn't that neat!

Calculations for T_H (Mug Handle Temperature):

Below is a section cut from the mug showing the liquid, wall thickness, and half of the handle. We want to determine the temperature at the end of the handle section, which we can compare to the Tmin of our simulation.





The handle was defined as an ellipse, as shown in the sketch that defines it. An ellipse was used so an Extend Distance and a Handle Height can have distinct values. The Handle Length (HL) shown is 1/4th of the ellipse. All we need to do is find the perimeter of the ellipse and divide by 4.

What the sigma! There is no formula for the perimeter of an ellipse?

The perimeter can be derived as in integral form, but it is an integral that cannot be solved. One has to resort to methods to determine an approximation.

It can be defined by an infinate series.

$$P = 2a\pi\left(1 - \sum_{n=1}^{\infty} \frac{(2n!)^2}{(2^n \cdot n!)^4} \cdot \frac{e^{2n}}{2n-1}\right)$$

Which can be expanded, as shown below. The first 8 terms are shown. What is nice with an expansion, is the fact that one can keep adding terms and looking at the difference between the last and previus result. If the different is within the accuracy one desires, the additions can be stopped. You can be happy by using the first 4 terms.

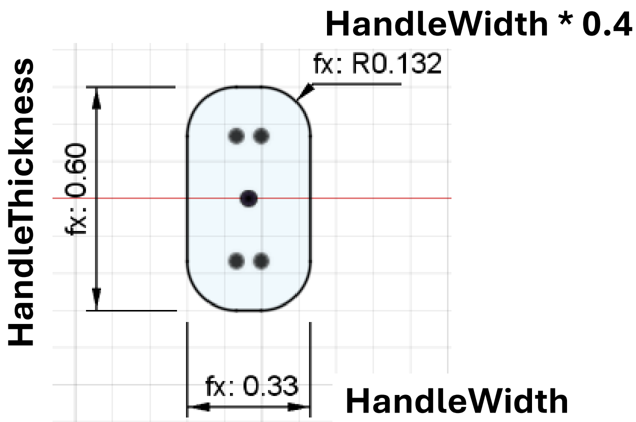
$$\frac{P}{\pi(a+b)} = 1 + \frac{h}{4} + \frac{h^2}{64} + \frac{h^3}{256} + \frac{25h^4}{16384} + \frac{49h^5}{65536} + \frac{441h^6}{2^{20}} + \frac{1089h^7}{2^{22}}$$

$$h = (a-b)^2/(a+b)^2$$

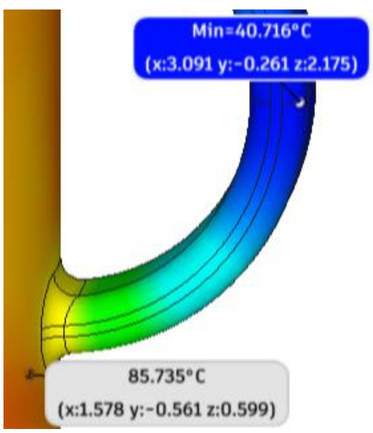
a = **HandleExtendDistance**

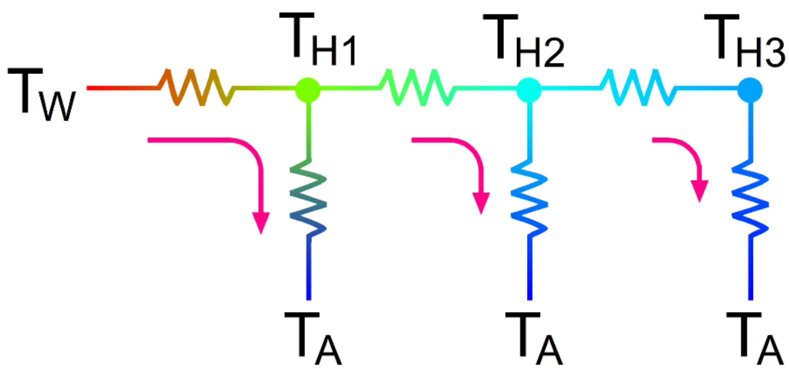
b = **HandleHeight / 2**

Next the cross-sectional area of the handle needs to be calculated. It was defined with the below sketch.



Thermal Circuit for Handle:





The handle has a large temperature gradient and a large surface area ratio to length and thus is best served by the shown circuit. Ideally, there would be an infinite number of resistors.

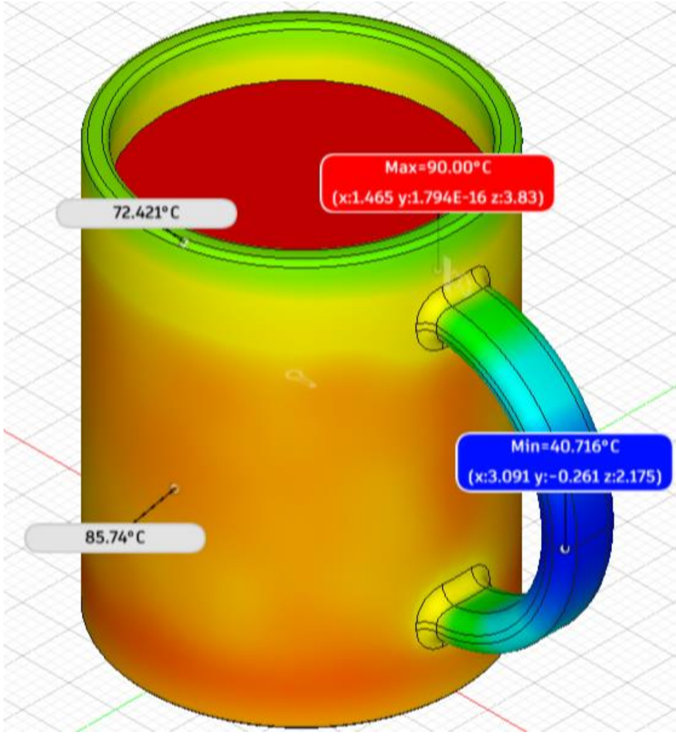
Hyperbolic cosine to the rescue!

The shape of a hanging cable, a catenary, is a hyperbolic cosine and this crazy function finds its place in various engineering calculations. It should be noted that the cables of a suspension bridge are closer to a parabola because the cables support a heavy road deck, which is a load distributed evenly along the x-axis.

One thing that is nice is at the end of the handle section **x = L** and thus **(1 – x/L) becomes 0** and thus the crazy top cosh function becomes **cosh(0)**, which equals **1** and thus the simpler form on the right can be used. The form on the right was also solved for T. If we were plotting the temperature, we would need the left form. You are welcome to do so for fun.

$$\frac{T - T_A}{T_S - T_A} = \frac{\cosh\left(\left(1 - \frac{x}{L}\right)\sqrt{\frac{hP}{kA}}L\right)}{\cosh\left(\sqrt{\frac{hP}{kA}}L\right)} \qquad T = \frac{\cosh(0)\,(T_S - T_A)}{\cosh\left(\sqrt{\frac{hP}{kA}}L\right)} + T_A$$

Compare your result to the minimum mug temperature, which should be the center of the handle.



Deliverables

1) screenshot of your simulation with a porcelain mug with 90 C water

2) screenshot of your simulation with a copper mug with 90 C water

3) screenshot of your simulation with a porcelain mug with 90 C water and radiational cooling off

4) calculation of wall temperature

5) calculation of handle temperature at its center

6) table showing comparison of simulated to calculated temperature for wall and handle center temperatures of the porcelain mug with radiational cooling off. Include the percentage difference for each comparison.

7) what was the handle temperature difference when radiational cooling was off? Which is the dominant cooling mechanism, convection or radiational?